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A LYAPUNOV APPROACH FOR SOSM BASED VELOCITY ESTIMATION AND ITS APPLICATION TO IMPROVE BILATERAL TELEOPERATION PERFORMANCE

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ABSTRACT

In many mechatronic applications, velocity estimation is required for implementation of closed loop control. Proportional-Integral control based differentiation has been proposed to estimate velocity in bilateral teleoperation. We propose a Second Order Sliding Mode (SOSM) based velocity estimation scheme for this application, since the SOSM approach is robust to small disturbances near the origin. Simulation results demonstrate the superior performance of the SOSM based velocity estimation over the PI-control approach for bilateral teleoperation in viscous environments. Additionally, a novel Lyapunov function based approach to stability analysis of the SOSM based differentiator is presented.

1 Introduction

Sliding mode control is widely used due to its attractive features of insensitivity to external and internal disturbances, accuracy and finite-time convergence. The approach is based on satisfaction of a desired constraint composed of the state variables and known as a sliding surface. Sliding mode control keeps the system on the sliding surface by high frequency switching of the control action. Sliding modes are obtained by introducing a discontinuous term in the control, designed such that the trajectories of the system are constrained to remain on the sliding surface. The resulting motion on the sliding surface is called a sliding mode.

Sliding mode order is defined in [1] by means of a smooth output function σ which is zero on the desired sliding surface. Then, provided that successive total time derivatives $\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}$ are continuous functions of the closed-loop system state variables, and the set $\sigma = \dot{\sigma} = \cdots = \sigma^{(r-1)} = 0$ is non-empty and consists locally of Fillipov trajectories [2], the motion on the set $\sigma = \dot{\sigma} = \cdots = \sigma^{(r-1)} = 0$ is called an r^{th} order sliding mode. The standard sliding mode control is of first order and although it is very accurate, robust and possesses the advantageous features mentioned previously, it suffers from a chattering phenomena. This chattering phenomena is caused by high frequency control switching which results in high frequency vibration of the controlled plant and may be detrimental to performance in some applications.

Second Order Sliding Mode (SOSM) control preserves the excellent features of the standard sliding mode control and if properly designed, can eliminate the chattering phenomena observed with standard sliding mode control. Levant [3] proposed a robust exact differentiator based on SOSM to estimate first order derivatives. The SOSM control based differentiator exhibits finite time convergence and can effectively deal with disturbances close to the origin of the error state space.

Further, we propose a novel Lyapunov function based approach to analyze the stability and convergence properties of this differentiator. The earlier approach proposed by Levant [3] was based on asymptotics and sufficient conditions resulting from a crude estimation were provided. In contrast, we propose a Lyapunov function based proof, giving more insight into the known properties of finite-time convergence and robustness to strong perturbations.

We propose use of the SOSM based differentiator to estimate velocity by real-time differentiation of position signals, leading to an improved bilateral teleoperation control performance. Designing a stable bilateral teleoperation control with

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minimal performance degradation in presence of variable communication delay is an important challenge [4], [5]. Various approaches have been suggested to achieve stable bilateral teleoperation under variable communication delay [6], and most of them require velocity input. In absence of direct velocity measurements, velocity is estimated from position input using real-time differentiators.

In related work, Hua and Liu [4] proposed use of a high gain observer based scheme to estimate velocity from the position signal for bilateral teleoperation without direct velocity measurements. High-gain observer based derivative estimation proposed by Vasiljevic [7], which may also be thought of as a Proportional-Integral (PI) control based differentiator is very effective in dealing with globally linearly growing disturbances, but near the origin, control authority becomes very weak due to its linear continuous structure. This results in poor performance in the presence of small disturbances close to the origin. In applications where high quality velocity estimations are required in presence of small disturbances, use of the SOSM control based differentiator exhibits distinct advantages over the PI control based differentiator. We show that when the remote environment is viscous, better teleoperation performance is obtained by using velocity estimates from the SOSM control based differentiator as compared to the PI control based differentiator.

The paper is organized as follows. In Section 2 we give the proposed Lyapunov proof of the SOSM control based differentiator and also describe the PI control based differentiator. In Section 3 we describe the bilateral teleoperation system and involved assumptions. Section 4 describes the controller design and gives the complete bilateral teleoperation control system. In Section 5, simulation model and various parameters choices are detailed. Section 6 presents the simulation results and discusses the contributions and limitations of the proposed approach. Finally we conclude the paper in Section 7.

2 Velocity Estimation

In the absence of direct velocity measurements, velocity estimation from real-time differentiation of position signals is needed to implement the controller for the bilateral teleoperation task under consideration.

In this study, we consider the following two differentiators for this task

1. Second Order Sliding Mode (SOSM) control based differentiator
2. Proportional-Integral (PI) control based differentiator

The PI control based differentiator was used by Hua and Liu [4] for estimating the velocity for bilateral teleoperation control under variable time-delay. We compute velocity from both the SOSM and the PI control based differentiators and compare their performance in bilateral teleoperation of viscous environments under variable time-delay. Both differentiators are described in the following subsections.

2.1 SOSM Control based differentiator

In order to differentiate an unknown signal $f(t)$, consider an auxiliary equation $\dot{x} = u$ where $x(t)$ is an estimate of the input signal $f(t)$, and the control law u is designed to drive the error in estimation $e(t) = x(t) - f(t)$ to zero. Levant [3] proposed a SOSM control scheme to drive $e(t)$ to zero and estimated the derivative $\dot{x}$ as

$$\dot{x} = -\lambda |e(t)|^{1/2} sign(e(t)) - \alpha \int_0^t sign(e(t)) \mathrm{d}t \tag{1}$$

where λ and α are positive constants. The SOSM based differentiator can be described in the equivalent state space form as

$$\begin{aligned} \dot{x}_1(t) &= -\lambda |x_1(t)|^{1/2} sign(x_1(t)) + x_2(t) \\ \dot{x}_2(t) &= -\alpha sign(x_1(t)) - \ddot{f}(t) \end{aligned} \tag{2}$$

where $x_1(t) = e(t) = x(t) - f(t)$ and $X = [x_1 \quad x_2]^T$.

The solutions of (2) are understood in the sense of Fillipov [2]. Fillipov solutions replace the differential equation (2) by a differential inclusion. A differential inclusion specifies that the state derivative belongs to a set of directions instead of a specific direction. A strong Lyapunov function is proposed to ensure the convergence of the state trajectories of system (2) to zero in finite time.

Consider the following function

$$\begin{aligned} V(x) &= 2\alpha |x_1| + \frac{1}{2} x_2^2 + \frac{1}{2} (\lambda |x_1|^{1/2} sign(x_1) - x_2)^2 \\ &= \zeta^T P \zeta \end{aligned} \tag{3}$$

where $\zeta = [|x_1|^{1/2} sign(x_1) \quad x_2]^T$ and

$$P = \frac{1}{2} \begin{bmatrix} 4\alpha + \lambda^2 & -\lambda \\ -\lambda & 2 \end{bmatrix}$$

We will show that this function is a Lyapunov function candidate for the system (2).

Lemma 1: The function $V(x)$ as given by (3) is a Lyapunov function candidate for the system (2) for $\alpha, \lambda > 0$.

Proof: If α and λ are positive, then P is a positive definite matrix and hence $V(x)$ is a positive definite function. Also since $\alpha > 0$, it is radially unbounded. The state trajectories $x(t) = \varphi(x_0, t)$ of the differential inclusion corresponding to the differential equation (2) are absolutely continuous functions and therefore $V(x)$ is a continuous function of time. $V(x)$ is not differentiable globally due to its lack of Lipschitzness at $x_1 = 0$. However $V(x)$ is continuously differentiable everywhere except on the surface $S = \{(x_1, x_2) \in \Re^2 | x_1 = 0\}$. It can be seen that the trajectories of (2) can cross S but cannot stay on it unless origin ($x_1 = 0, x_2 = 0$) has been reached and $\ddot{f}(t) = 0$. This means that $V(x)$ is differentiable for almost every t, and on those points derivative can be calculated in the usual way. Thus $V(x)$ is a valid Lyapunov function candidate and Lyapunov's theorem can be applied by just considering the points where $V(x)$ is differentiable. □

Theorem 1: If the derivative of $f(t)$ exists with a Lipschitz's constant C and the gains α and λ satisfy

$$\lambda > 0, \quad \alpha > 3C + \frac{2C^2}{\lambda^2} \tag{4}$$

then the origin of (2) is globally asymptotically stable after a finite-time transient process.

Proof: Choosing (3) as the Lyapunov function candidate, its time derivative along the solution trajectories of (1) is given as

$$\dot{V} = -\frac{1}{|e|^{1/2}}\zeta^T Q\zeta - \ddot{f}(t)q^T\zeta$$

where $q = [-\lambda \quad 2]$ and

$$Q = \frac{\lambda}{2}\begin{bmatrix} 2\alpha + \lambda^2 & -\lambda \\ -\lambda & 1 \end{bmatrix}$$

Now by definition of Lipschitz's constant, we have $|\ddot{f}(t)| \leq C$. Using this bound on $|\ddot{f}(t)|$, it can be shown that

$$\dot{V} \leq -\frac{1}{|e|^{1/2}}\zeta^T\tilde{Q}\zeta \leq -\frac{1}{|e|^{1/2}}\lambda_{min}\{\tilde{Q}\}\|\zeta\|_2^2 \tag{5}$$

where

$$\tilde{Q} = \frac{\lambda}{2}\begin{bmatrix} \lambda^2 + 2(\alpha - C) & -(\lambda + 2C) \\ -(\lambda + 2C) & 1 \end{bmatrix}$$

and $\lambda_{min}\{\chi\}$ is the minimum eigenvalue of χ.

If the gains satisfy the condition (4), then it is easy to see that $\tilde{Q} > 0$ hence $\dot{V}$ is a negative definite function.

From (3), we can write

$$\lambda_{min}\{P\}\|\zeta\|_2^2 \leq V(x) \leq \lambda_{max}\{P\}\|\zeta\|_2^2 \tag{6}$$

where $\|\zeta\|_2^2 = |x_1| + x_2^2$ is the Euclidian norm of ζ. Also observe that

$$|e|^{1/2} \leq \|\zeta\|_2 \leq \frac{V^{1/2}}{\lambda^{1/2}P} \tag{7}$$

It follows from (5),(6) and (7), that

$$\dot{V} \leq -\gamma V^{1/2}(x)$$

where

$$\gamma = \frac{\lambda_{min}^{1/2}\{P\}\lambda_{min}\{\tilde{Q}\}}{\lambda_{max}\{P\}}$$

Since the solution of the differential equation

$$\dot{\upsilon} = -\gamma\upsilon^{1/2}, \upsilon(0) = \upsilon_0 \geq 0$$

is given by

$$\upsilon(t) = \left(\upsilon^{1/2} - \frac{\gamma}{2}t\right)^2 \tag{8}$$

it follows from the comparison principle [8] that $V(t) \leq \upsilon(t)$ when $V(x_0) \leq \upsilon(x_0)$. From (8) we obtain that $V(x)$ and $x(t)$ converge to zero in finite time and reaches that value at most after $T = \frac{2V^{1/2}(x_0)}{\gamma}$ □

Levant [3] gave a sufficient condition for the convergence of $u(t)$ to $\dot{f}(t)$ given as

$$\alpha > C\,,\ \lambda^2 \geq 4C\frac{\alpha + C}{\alpha - C} \tag{9}$$

It can be observed that the proposed condition (4) is more relaxed at least in the choice of the gain λ.

2.2 PI Control based differentiator

Following the same differentiator construction as in the previous subsection, consider $x(t)$ an estimate of the input signal $f(t)$. Define error in the estimate as $e(t) = x(t) - f(t)$, then the first order derivative can be estimated by replacing u with a Proportional Integral(PI) control law as following

$$\dot{x} = -\left(\frac{K_p}{\varepsilon}\right)e(t) - \left(\frac{K_i}{\varepsilon^2}\right)\int_0^t e(t)\,\mathrm{d}t \tag{10}$$

Lemma 2: For the system (10), error $e(t)$ can rendered sufficiently small by adjusting the parameter ε.

Proof: Consider the state variable $X = [x_1, x_2]^T$ defined as $x_1 = e$ and $x_2 = \varepsilon e$ where ε is a small positive parameter, system (10) can be written as

$$\begin{aligned} \varepsilon\dot{x}_1 &= -K_p x_1 + x_2 \\ \varepsilon\dot{x}_2 &= -K_i x_1 - \varepsilon^2\ddot{f}(t) \end{aligned} \tag{11}$$

The above equation can be represented in matrix notation as

$$\varepsilon\dot{X} = AX + B\eta \tag{12}$$

where $A = \begin{bmatrix} -K_p & 1 \\ -K_i & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ and $\eta = -\varepsilon^2\ddot{f}(t)$. The parameters K_p and K_i are chosen such that A is Hurwitz stable.

Choose the Lyapunov function for system (12) as

$$V = \varepsilon X^T PX \tag{13}$$

where P is a positive definite matrix satisfying $PA + A^T P = -Q < 0$. Taking the derivative of V along the solution trajectories of the system (12)

$$\begin{aligned} \dot{V} &= -X^T QX + 2X^T PB^T\eta \\ &\leq -X^T(Q - \gamma^{-2}PBB^T P)X + \gamma^2\eta^2 \end{aligned} \tag{14}$$

where γ is a positive scalar such that $Q - \gamma^{-2}PBB^T P > 0$. From (14) we obtain

$$\begin{aligned} \|X(t)\| &\leq \frac{\gamma \sup|\eta|}{\sqrt{\lambda_{min}(Q - \gamma^{-2}PBB^T P)}} \\ &\leq \frac{\gamma\varepsilon^2 C}{\sqrt{\lambda_{min}(Q - \gamma^{-2}PBB^T P)}} \end{aligned} \tag{15}$$

as $t \to \infty$, where $\lambda_{min}(\chi)$ denotes the minimum eigenvalue of the matrix χ. Parameter ε can be chosen small to obtain a small bound on error e. It can be observed from (14) that as $\varepsilon \to 0$ the system converges exponentially and the size of the error bound is determined by the choice of ε.

3 Bilateral Teleoperation System

A standard bilateral teleoperation system as shown in Fig. 1 is used to compare the performance of both velocity estimation schemes. The master and slave are n-link serial manipulators consisting of only revolute joints described by the following equations

$$\begin{aligned} D_m(q_m)\ddot{q}_m + C_m(\dot{q}_m, q_m)\dot{q}_m &= \tau_m - J_m^T F_h \\ D_s(q_s)\ddot{q}_s + C_s(\dot{q}_s, q_s)\dot{q}_s &= \tau_s + J_s^T F_e \end{aligned} \tag{16}$$

where $q_m(t), q_s(t) \in R^n$ are the joint displacement vectors, $\dot{q}_m, \dot{q}_s \in R^n$ are the vectors of joint velocities, $D_m(q_m)$ and $D_s(q_s)$ are inertia matrices of the master and slave robot manipulators respectively, $C_m(\dot{q}_m, q_m)\dot{q}_m$ and $C_s(\dot{q}_s, q_s)\dot{q}_s$ are the vectors of centripetal and coriolis torques, J_m and J_s are jacobian matrices of the master and slave manipulators, F_h and F_e are forces applied by the human operator and environment, and τ_m and τ_s are the applied torques computed by the master and slave controllers.

The following assumptions are made regarding the bilateral teleoperation system under consideration

Assumption 1: The inertia matrices of the master and slave manipulators are uniformly bounded, that is the inequality $0 < \mu_{j1}I \leq D_j(q_j) \leq \mu_{j2}I < \infty$ holds true where μ_{j1} and μ_{j2} are positive constants, $j = m, s$.

Assumption 2: The forward and backward time delays in the communication channel, $T_1(t)$ and $T_2(t)$ are bounded such that $T_1(t) \leq \overline{T_1}$ and $T_2(t) \leq \overline{T_2}$.

The inertia matrices $D_m(.), D_s(.)$ and centripetal and coriolis torque matrices $C_m(.), C_s(.)$ are considered to be completely unknown and the joint velocity measurements are also unavailable. Joint velocities are required for the controller design discussed in Section 4 and are estimated using the differentiators described in Section 2.

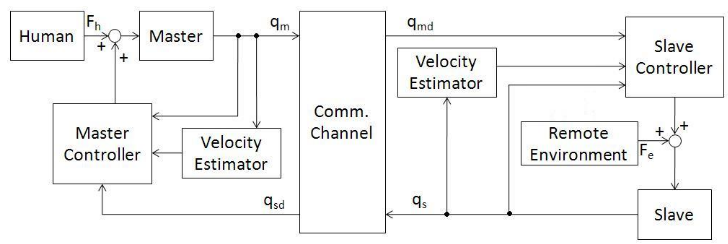

Figure 1. Bilateral teleoperation system with velocity estimation.

4 Controller Design

This section details the controller design for system (16). The control structure proposed by Hua and Liu [4] is used, but instead of estimating velocity with a linear PI control based differentiator, we propose the use of a SOSM control based differentiator, thereby improving the teleoperation performance for the given task.

The controller is described as follows

$$\begin{aligned} \tau_m &= -k_m(q_m - q_s(t - T_1(t))) - \alpha_m z_{m2} \\ \tau_s &= k_s(q_m(t - T_2(t)) - q_s) - \alpha_s z_{s2} \end{aligned} \tag{17}$$

where $T_1(t)$ is the delay in the forward path from master to slave and $T_2(t)$ is the delay in the backward path from slave to master, k_m and k_s are the proportional gains, α_m and α_s are damping gains, and z_{m2} and z_{s2} are estimated velocity signals of the master and slave manipulators.

Combining the controller with the system (16), the complete bilateral teleoperation controlled system is obtained as

$$
\begin{aligned}
&D_m(q_m)\ddot{q}_m + C_m(\dot{q}_m, q_m)\dot{q}_m \\
&= -k_m(q_m - q_s(t - T_1(t))) - \alpha_m z_{m2} - J_m^T F_h \\
&D_s(q_s)\ddot{q}_s + C_s(\dot{q}_s, q_s)\dot{q}_s \\
&= k_s(q_m(t - T_2(t)) - q_s) - \alpha_s z_{s2} + J_s^T F_e
\end{aligned} \tag{18}
$$

From the main result of [4], the closed loop system (18) is stable, and the velocities $\dot{q}_m$, $\dot{q}_s$ and position tracking error $(q_m - q_s)$ are bounded if there exists positive scalars ω_1, ω_2 and positive definite matrices Z, S such that the following LMI is satisfied

$$
\Psi = \begin{bmatrix} \Psi_{(1,1)} & 0 & \Psi_{(1,3)} & \Psi_{(1,4)} \\ * & \Psi_{(2,2)} & \Psi_{(2,3)} & \Psi_{(2,4)} \\ * & * & \Psi_{(3,3)} & 0 \\ * & * & * & \Psi_{(4,4)} \end{bmatrix} < 0 \tag{19}
$$

where $*$ represents the transpose of the corresponding matrix, and the elements are as follows

$$
\begin{aligned}
\Psi_{(1,1)} &= -2\alpha_m I + 2\mu_{m2}\varepsilon_1 I + \overline{T}_2 k_m^2 S^{-1} \\
\Psi_{(1,3)} &= -\alpha_m \varepsilon_1 I - k_m I + \overline{T}_2 \varepsilon_1 k_m^2 S^{-1} + kI \\
\Psi_{(1,4)} &= [\overline{T}_1 I \quad I \quad I \quad 0] \\
\Psi_{(2,2)} &= \omega_1 \varepsilon_1^2 \mu_{m2}^2 I + \omega_2 \varepsilon_2^2 \mu_{s2}^2 I + 2\mu_{s2}\varepsilon_2 I - 2\alpha_s I + \overline{T}_1 k_m^2 Z^{-1} \\
\Psi_{(2,3)} &= k_s I + \alpha_s \varepsilon_2 I - \overline{T}_1 k_s^2 \varepsilon_2 Z^{-1} - kI \\
\Psi_{(2,4)} &= [0 \quad 0 \quad 0 \quad \overline{T}_2 I] \\
\Psi_{(3,3)} &= -2k_m \varepsilon_1 I - 2k_s \varepsilon_2 I + \overline{T}_2 k_m^2 \varepsilon_1^2 S^{-1} + \overline{T}_1 k_s^2 \varepsilon_2^2 Z^{-1} \\
\Psi_{(4,4)} &= diag\{-\overline{T}_1 Z^{-1}, -\omega_1 I, -\omega_2 I, -\overline{T}_2 S^{-1}\}
\end{aligned}
$$

k,ε_1 and ε_2 are positive scalars satisfying

$$
k \geq \mu_{m1}^{-1}(\mu_{m2}\varepsilon_1)^2 + \mu_{s1}^{-1}(\mu_{s2}\varepsilon_2)^2 \tag{20}
$$

The LMI toolbox of MATLAB was used for solving (19). The LMI (19) contains the controller parameters k_m, k_s, α_m, α_s and the bounds on forward and backward time delays $\overline{T}_1$ and $\overline{T}_2$. For any given choice of controller parameters, fixing the maximum bound on forward time delay can give the maximum bound on the backward time delay and vice versa. Also for given bounds on the time delays, controller parameters can be chosen such that the condition (19) is satisfied.

5 Simulation

The bilateral teleoperation control system (18) is simulated with 2-link serial manipulators as the master and slave devices and velocity estimated using the SOSM control and the PI control based differentiator. The manipulators are described by the following equations

$$
\begin{aligned}
D_m(q_m)\ddot{q}_m + C_m(\dot{q}_m, q_m)\dot{q}_m &= \tau_m - J_m^T F_h \\
D_s(q_s)\ddot{q}_s + C_s(\dot{q}_s, q_s)\dot{q}_s &= \tau_s + J_s^T F_e
\end{aligned} \tag{21}
$$

where

$$
D_m(q) = D_s(q) = \begin{bmatrix} D_{11} & D_{12} \\ * & D_{22} \end{bmatrix},
$$

$$
C_m(q,\dot{q}) = C_s(q,\dot{q}) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix},
$$

$$
J_m(q) = J_s(q) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix},
$$

$$
\begin{aligned}
D_{11} &= m_1 l_{c1}^2 + m_2(l_1^2 + l_{c1}^2 + 2l_1 l_{c2}\cos q_2) + I_1 + I_2 \\
D_{12} &= m_2(l_{c2}^2 + l_1 l_{c2}\cos q_2) + I_2, \quad D_{22} = m_2 l_{c2}^2 + I_2 \\
C_{11} &= -m_2 l_1 l_{c2}\sin q_2 \dot{q}_2, \quad C_{12} = -m_2 l_1 l_{c2}\sin q_2(\dot{q}_1 + \dot{q}_2) \\
C_{21} &= m_2 l_1 l_{c2}\sin q_2 \dot{q}_1, \quad C_{22} = 0 \\
J_{11} &= -l_1\sin q_1 - l_2\sin(q_1 + q_2), \quad J_{12} = -l_2\sin(q_1 + q_2) \\
J_{21} &= l_1\cos q_1 + l_2\cos(q_1 + q_2), \quad J_{22} = l_2\cos(q_1 + q_2)
\end{aligned}
$$

The parameters used for the simulation are as follows: $m_1 = 5kg$, $m_2 = 1kg$, $l_1 = 0.7m$, $l_2 = 0.5m$, $l_{c1} = 0.35m$, $l_{c2} = 0.25m$, $I_1 = 0.5kg.m^2$ and $I_2 = 0.5kg.m^2$. The user exerts a force F_h in the Y-direction at the end-effector of the master. the force is zero at $t = 0$s and increases linearly to 5 N over a period of 10s after which it is held constant at 5 N. The soft viscous environment is a spring-damper system situated at $y = 0.2m$ and the feedback force is given as

$$
F_e = \begin{cases} K_e(y - 0.2) + B_e\dot{y}, & if \;\; y > 0.2 \\ 0, & if \;\; y \leq 0.2 \end{cases} \tag{22}
$$

where $K_e = 100N/m$ and $B_e = 10N.s/mm$. These values are chosen such that the environment impedance lies in the typical impedances range observed in biological tissue palpation tasks [9].

The controller parameters are chosen as $k_m = k_s = 1$ and $\alpha_m = \alpha_m = 10$. We consider two cases: first with velocity estimated using the SOSM control based differentiator and secondly with the PI control based differentiator. The parameters for the SOSM based differentiator are chosen as $\alpha = 1, \lambda = 0.3$ for the master side and $\alpha = 16, \lambda = 8$ for the slave side. Note that these gains satisfy the condition (4) obtained from the proposed Lyapunov approach but violate the sufficient condition (9) proposed in [3]. Identification of such gains demonstrates that the gain conditions obtained from the Lyapunov approach extend the choice of gain pairs given by the sufficient condition proposed in [3]. Parameters for the PI control based differentiator are chosen as $K_p = 0.5$, $K_i = 0.1$ and $\varepsilon = 0.01$ for both master and slave.

Choosing the bound of forward time delay as $\overline{T}_1 = 1s$, by solving the LMI (19) using the LMI Optimization toolbox of MATLAB for the chosen controller parameters we obtain the maximum allowable bound for the backward time delay as $\overline{T}_2 = 7.8167s$. With these bounds on the time delays, we choose $T_1(t) = 0.2\sin(t) + 0.8$ and $T_2(t) = 2.8\sin(t) + 5$ seconds.

6 Results and Discussion

In this section we present the results obtained from the simulations with the SOSM and the PI control based differentiators described in Section 5. The results shows improved teleoperation performance in soft and viscous environments with velocity estimated using the SOSM control based differentiator. Figures 2 and 3 show the joint velocities estimated by the PI control based and the SOSM control based differentiators. It is observed that

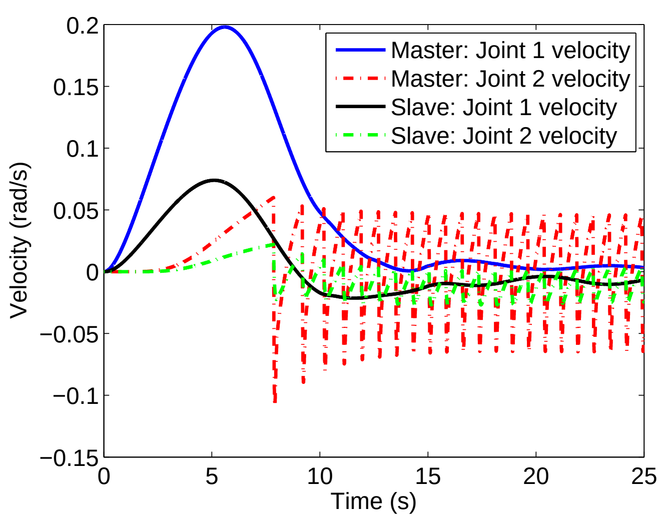

Figure 2. The joint velocities of master and slave estimated with the PI control based differentiator. Oscillatory behavior is observed during viscous interaction.

the PI control based differentiator exhibits sustained oscillations in the presence of disturbances resulting from viscous interaction, whereas velocities estimated by the SOSM control based differentiator remain unaffected. This difference in performance in velocity estimation manifests itself in the position plot of the

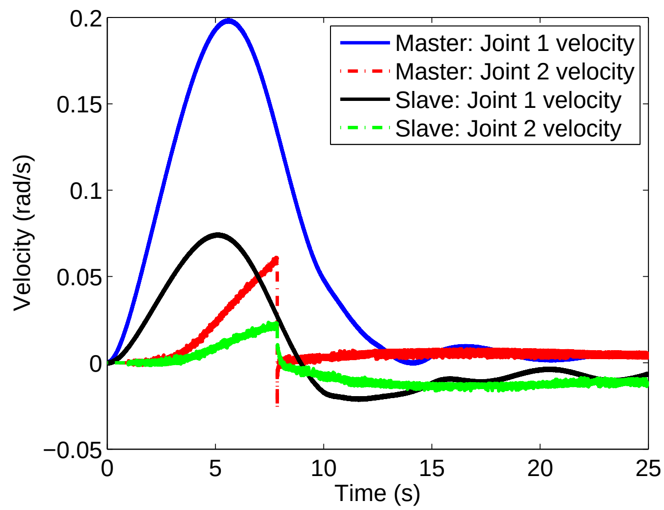

Figure 3. The joint velocities of master and slave estimated with the SOSM control based differentiator. The velocities quickly converge to a steady state value during viscous interaction.

master and slave end-effectors as shown in Fig. 4. The slave end-effector position plot shows chatter during viscous interaction due to poor velocity estimation by the PI control based differentiator. On the other hand, with velocity estimated by the SOSM control based differentiator the chatter is absent and a stable interaction is observed.

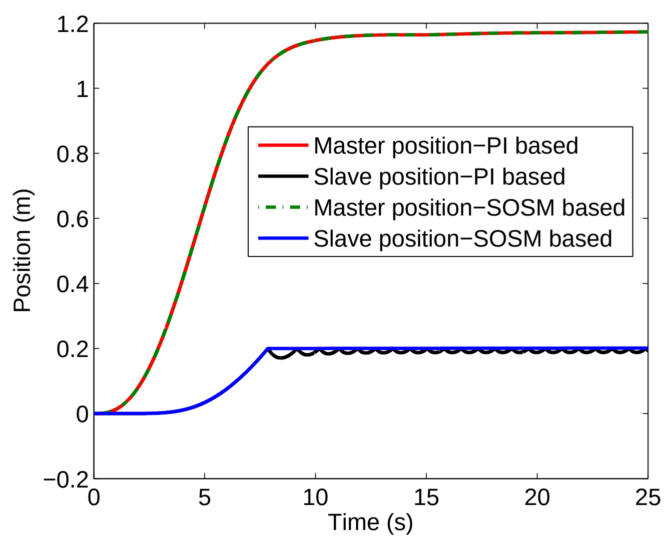

Figure 4. Plots of y-position of the master and slave end-effector for velocity estimated by the PI control and the SOSM control based differentiator. Chatter is observed in the slave side with the PI Control based differentiator due to errors in velocity estimation.

The chatter at the slave side with the PI control based differentiator as compared to the relatively smooth behavior with the

SOSM control based differentiator is due to differences in their structure (linear & continuous vs. nonlinear & discontinuous) and convergence properties. The PI control based differentiator is linear in error $e(t)$ (11) and hence the control authority becomes small when $e(t)$ is close to zero. The parameter ε could be chosen small to set the PI control gains high so that control effort is still able to deal with small disturbances but that will also amplify any noise in the signal. The SOSM control based differentiator is non-linear and discontinuous at $e(t) = 0$, which means that even when error is very close to zero a strong control action is generated. In the case of viscous interaction, the disturbances are small in magnitude and are proportional to the velocity. Hence the SOSM control based differentiator provides better velocity estimate than the linear PI control based differentiator. Also, with the SOSM control based differentiator, finite-time convergence is achieved as long as the condition (4) is satisfied, whereas with the PI control based differentiator convergence is asymptotic as $e(t) \to 0$ as $\varepsilon \to 0$. Due to finite time convergence property, the catching up time can be made small by proper choice of gains but with the PI control based differentiator setting ε to zero is not possible hence only convergence to a bound is obtained as given by equation (15).

7 Conclusion

In this paper we present a novel Lyapunov approach for analyzing the stability and convergence properties of the Second Order Sliding Mode (SOSM) control based differentiator. We propose the use of the SOSM control based differentiator for velocity estimation by real-time differentiation of position signals, and test its effectiveness in bilateral teleoperation of viscous remote environments. Simulations are performed to demonstrate improvement in teleoperation performance with proposed SOSM control based velocity observer over the linear velocity observer.

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