

A Method for Selecting Velocity Filter Cut-Off Frequency for Maximizing Impedance Width Performance in Haptic Interfaces

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This paper analyzes the effect of velocity filtering cut-off frequency on the Z-width performance of haptic interfaces. Finite difference method (FDM) cascaded with a low pass filter is the most commonly used technique for estimating velocity from position data in haptic interfaces. So far, there is no prescribed method for obtaining the FDM + filter cut-off frequency that will maximize Z-width performance. We present a simulation based method to demonstrate that there exists such an ideal FDM + filter cut-off frequency, and that it can be predicted by numerical simulation based on an identified model of a haptic interface. Experiments are conducted on a single degree-of-freedom (DOF) linear haptic interface to validate the simulation results. [DOI: 10.1115/1.4028526]

1 Introduction

Impedance width, or Z-width, is the dynamic range of impedances that can be passively rendered by a haptic interface. Z-width was proposed by Colgate and Brown [1] as a fundamental measure of performance for haptic interfaces. A larger achievable dynamic range of impedances translates to a more realistic haptic rendering of a virtual environment. Therefore, it is desirable to maximize the Z-width boundary of a haptic display. The Z-width boundary is dependent on various parameters of the haptic interface such as the device dynamics [2], sampling frequency [3], position sensor quantization [1,4,5], actuator saturation [6], time delays [7] and velocity estimation [8,9]. This study investigates the effect of velocity filter cut-off frequency on the Z-width performance. Specifically, numerical simulations are conducted that are supported by experiments to demonstrate that there is an ideal

velocity filter cut-off frequency which maximizes the Z-width performance of a haptic interface, and its value can be predicted by simulation.

In haptics applications, velocity is typically estimated from the position encoder data using the FDM, or equivalently the backward difference method for real time implementation. The velocity obtained by FDM is very noisy due to sampling and quantization effects, and a low pass filter is required to smooth the signal [1]. Choice of the FDM + filter cut-off frequency determines the phase distortion introduced in the velocity signal, and the amount of noise present after filtering. This trade-off between time-delay and noise in velocity signals suggests that there is a dependence of Z-width performance on the FDM + filter cut-off frequency, since the accuracy of velocity estimation directly affects the Z-width performance [8]. Despite its ubiquitous use for velocity estimation, few researchers have specifically addressed the effect of FDM + filter cut-off frequency selection on Z-width performance.

In many haptics applications, the cut-off frequency in the FDM + filter velocity estimation method is either chosen in an ad-hoc fashion [1,10] or is manually tuned for a given application [11]. Several researchers have used FDM + filter as a benchmark in comparing advanced velocity estimation methods, but the FDM + filter cut-off frequency was chosen in an ad hoc fashion [9,12–14]. Díaz et al. proposed that FDM + filter cut-off frequency be chosen in range between the bandwidth of the human hand (~10 Hz, [15]) and the first vibration mode of the haptic interface device [7]. This method still leaves ambiguity in selection of the cut-off frequency, which is commonly chosen near the allowable lower bound.

In this paper, the effect of FDM + filter cut-off frequency on Z-width performance of a haptic display is studied by conducting linear stability analysis, numerical simulations and experimental analysis. It is found that there is an “ideal” cut-off frequency which maximizes the Z-width performance of a haptic device, and its value can be predicted by numerical simulation. The simulation is conducted using a linear device model obtained by performing system identification of the haptic interface, and considering the actuator saturation and position quantization nonlinearities. The ideal FDM + filter cut-off frequency and the variation of Z-width performance with varying cut-off frequencies predicted by the simulation is validated experimentally. It is found that the linear stability analysis is not sufficient to predict the ideal FDM + filter cut-off frequency, as position sensor quantization plays an important role in determination of the ideal cut-off frequency. We show that numerical simulation considering the quantization nonlinearity provides an effective way of identifying the FDM + filter cut-off frequency that will maximize the Z-width performance in a haptic device.

2 Methods to Explore Effect of Velocity Filter Cut-Off Frequency on Z-Width Performance

2.1 Linear Analysis. Linear stability analysis of a single-DOF linear impedance type haptic interface device is employed to study the effect of velocity low pass filter cut-off frequency on the Z-width of haptic devices. The analysis is based on the approach proposed by Gil et al. [16]. Consider a single-DOF haptic interface represented with a mass-spring-damper model $G(s) = 1/(m_{eq}s^2 + b_{eq}s + k_{eq})$ and velocity estimated using FDM + second-order Butterworth low pass filter, as shown in Fig. 1. m_{eq} , b_{eq} , and k_{eq} represent the effective mass, damping, and stiffness of the haptic device. With an aim to minimize delay in estimations, filter-order is limited to second-order. The continuous time transfer function of a second-order low pass Butterworth filter is given as

$$H(s) = \frac{H_0\omega_c^2}{s^2 + 1.414s\omega_c + \omega_c^2} \quad (1)$$

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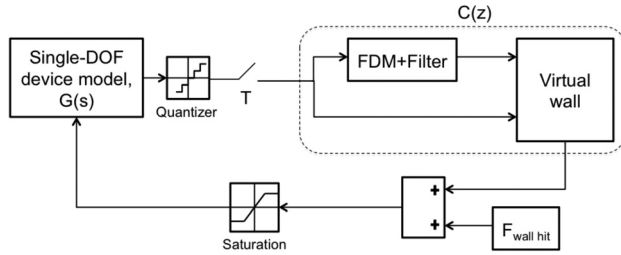


Fig. 1 Schematic showing the simulation of the single-DOF haptic interface device with velocity estimated using FDM + filter

where H_0 is the DC gain and ω_c is the cut-off frequency. We used the bilinear transform to obtain the discrete time transfer function ($H(z)$) of the low pass second-order Butterworth filter with sampling period T . Bilinear transformation is used because it preserves the causal, stable nature of the continuous time system while mapping to the discrete domain.

The virtual environment is modeled as a spring-damper virtual wall $C(z) = K + BH(z)(1 - z^{-1})/T$ with virtual wall stiffness (K) and virtual wall damping (B). The force applied by the human operator is f_h , and x_h is the position output of the haptic device.

The discrete time closed-loop transfer function is given by

$$x_h = \frac{Z[Gf_h]}{1 + Z[\text{ZOHG}]C(z)} \quad (2)$$

where Z is the Z-transform operator and ZOH is the zero-order-hold. The characteristic equation is given as

$$1 + Z[\text{ZOHG}]C(z) = 0 \quad (3)$$

The characteristic equation can be used to find the stability boundary

$$1 + Z[\text{ZOHG}]\left(K + BH(z)\frac{1 - z^{-1}}{T}\right) = 0 \quad (4)$$

or

$$1 + K\left(\frac{Z[\text{ZOHG}]}{1 + Z[\text{ZOHG}]BH(z)\frac{1 - z^{-1}}{T}}\right) = 0 \quad (5)$$

The stability region is determined by the (K, B) pairs which satisfy the inequality

$$K < \text{GM}\left(\frac{Z[\text{ZOHG}]}{1 + Z[\text{ZOHG}]BH(z)\frac{1 - z^{-1}}{T}}\right) \quad (6)$$

where $\text{GM}(\cdot)$ is the gain margin.

The device parameters were specified as $m_{eq} = 0.4160$ kg, $b_{eq} = 5.5195$ N s/m and $k_{eq} = 11.0626$ N/m. For a particular cut-off frequency ω_c , virtual wall damping B is varied and the corresponding values of K satisfying the inequality (6) constitute the “uncoupled stability” boundary. Uncoupled stability does not consider the coupling of human operator with the haptic device, resulting a larger region of stability encompassing the Z-width [17]. The plot of maximum values of K satisfying the inequality (6) for varying B gives the uncoupled stability boundary. The approach followed in conducting the linear analysis assumes that for chosen device parameter values, GM is well defined for the range of ω_c and B considered. If these parameters are allowed to take any arbitrary value, then it is possible to have multiple crossings of phase at -180 deg and GM will not be well defined. In that case above approach is not applicable.

Figure 2 shows the uncoupled stability boundary plots obtained by the linear analysis for varying cut-off frequencies. It is observed that with increasing cut-off frequencies, the boundary increases. It should be noted that the Z-width boundary will always be smaller than the uncoupled stability boundary, and in practice will be further limited due to actuator saturation, quantization, friction and other nonlinearities.

2.2 Simulation Analysis Considering Nonlinearities. The linear analysis in Sec. 2.1 did not take into account some important nonlinearities such as actuator saturation and quantization. Actuator selection depends on the target application and design requirements of the haptic device, and will dictate the actuator saturation limits. Position sensor quantization, on the other hand, is something that can be selected independently of the hardware design requirements and is very important for the accuracy of the haptic device control. Position sensor quantization directly affects the velocity estimation, and consequently the Z-width performance. We performed system identification of a single-DOF haptic interface device, and used the identified model in the simulation for estimating the Z-width boundaries, and studied the effect of velocity filter cut-off frequency on Z-width performance.

2.2.1 Single-DOF Haptic Interface. The Z-width experiments were performed on a custom built single DOF, linear, impedance type haptic interface device. The device is shown in Fig. 3. The haptic interface has a workspace of 0.15 m and the handle position is measured by a linear incremental encoder with a resolution of $1 \mu\text{m}$. A detailed description and technical specifications of the device can be found in Ref. [8].

2.2.2 System Identification and Simulation Model. A physics-based model of the single-DOF haptic interface device was developed, given as

$$m_{eq}\ddot{x} + b_{eq}\dot{x} + k_{eq}(x - x_0) = k_{DAQ}V_{in} \quad (7)$$

where x is the handle position, m_{eq} is the equivalent mass of the system comprising of the cart mass and motor inertia, b_{eq} is equivalent physical damping in the system, k_{eq} is the equivalent stiffness of the system arising from the capstan drive dynamics [18], and x_0 is the equilibrium position for this equivalent spring. The equivalent stiffness k_{eq} of the system arises from difference between tensions due to preloading in the two segments of the capstan cable which varies during the range of motion of the slider, and equate to zero at the equilibrium point x_0 . This difference in preload tensions is usually very small and ignored in

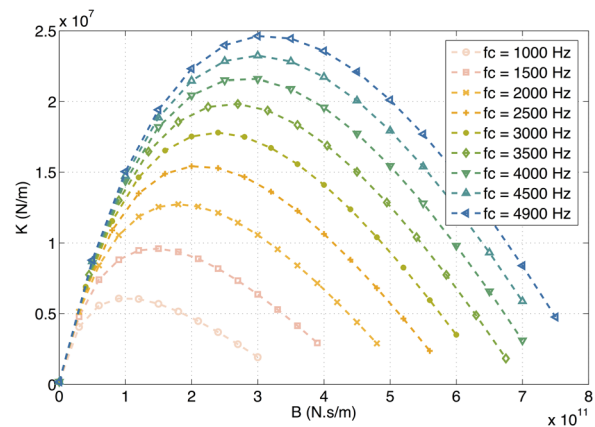


Fig. 2 Z-width plots are obtained analytically for a single-DOF haptic device with velocity estimated using FDM + filter. The sampling frequency is fixed at 10 kHz and the plots are generated for varying cut-off frequencies ($f_c = \omega_c/2\pi$).

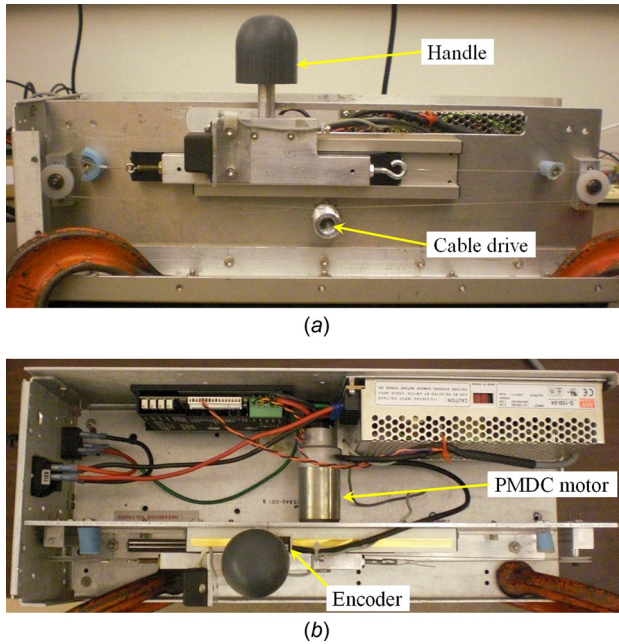


Fig. 3 A single-DOF haptic device is used as the experimental setup. (a) Front view and (b) top view.

modeling of most haptic interfaces, but appears in modeling of our system due to very low friction. k_{DAQ} is the gain relating the voltage input to the force output of the motor + capstan drive. Gray-box system identification was performed in the time domain to identify the parameter values. k_{DAQ} was estimated as $k_{DAQ} = k_t k_{amp} / r_c$, where k_t is the torque constant of the motor, k_{amp} is the voltage-to-current amplifier gain, and r_c is the radius of the capstan drum. The average parameter values were identified as $m_{eq} = 0.4160$ kg, $b_{eq} = 5.5195$ Ns/m, $k_{eq} = 11.0626$ N/m, $x_0 = 0.0517$ m and $k_{DAQ} = 0.5906$ N/V.

The schematic of the simulation model is shown in Fig. 1. The single-DOF haptic interface device was modeled as Eq. (7). A hybrid simulation model was constructed using a continuous solver to simulate the single-DOF device and a fixed step solver to simulate the discrete time control. The virtual wall was simulated at 10 kHz and the actuation rate was fixed at 1 kHz. The actuation rate is chosen smaller than the control loop rate because in the experimental setup a pulse width modulation (PWM) amplifier with a switching frequency of 36 kHz is used, and for an actuation rate of 10 kHz, the PWM amplifier is unlikely to run the current control loop sufficiently fast. The saturation limits imposed by the data acquisition card are ± 10 V. MATLAB® and SIMULINK® were used to perform the simulation.

2.2.3 Protocol for Estimating Z-Width. We simulated the experimental protocol adopted in our previous work [8] to estimate the Z-width boundary. At the beginning of the experiment, the handle was placed 0.07 m away from the virtual wall. A constant torque was commanded to the motor resulting in an effective force of 2.835 N at the handle, driving it toward the wall. After allowing a 4 s period for the wall hit to reach steady state, mean position was recorded. Root mean square (RMS) difference between the mean position and the instantaneous position of the handle was computed for the next 2 s, and if this RMS difference was smaller than a threshold ε , then there were no sustained oscillations present at the steady state. A wall hit was declared stable if there were no sustained oscillations at the steady state. We chose $\varepsilon = 1.5 \times 10^{-5}$ m for our experiments. Although the specific value of ε affects the size of the Z-width boundary, it does not affect the qualitative dependence of the Z-width performance on the velocity cut-off frequency. We started with a nominal value of virtual wall stiffness (K) and virtual wall damping (B) for which the wall hit

was stable. K was increased in steps until an unstable wall hit was observed, at which point B was incremented by a step. If the wall hit was stable after incrementing B , then K was incremented until an unstable wall hit was observed, else K was decremented until a stable wall hit was observed. B was again incremented and the whole cycle repeated until the virtual wall was not stable for any value of K . The plot of K versus B for which the virtual wall was marginally stable constitutes the Z-width boundary.

Figure 4 shows the Z-width plots obtained by simulation. It is observed in Fig. 4(a) that for a position quantization of $1 \mu\text{m}$, the Z-width performance saturates after initially increasing monotonically with increasing cut-off frequency until around $f_c = 2000$ Hz. No significant increase in Z-width performance is observed after $f_c = 2000$ Hz. On doubling the position quantization to $2 \mu\text{m}$, the saturating trend observed earlier changes and the Z-width performance begins to decrease after reaching a peak at $f_c = 1700$ Hz, as shown in Fig. 4(b).

2.3 Experimental Analysis. Experiments were conducted on the single-DOF device to observe the effect of changing velocity filter cut-off frequency on the Z-width performance, and validate the predictions of the simulation analysis. The automated wall hit task protocol described in Sec. 2.2.3 was used to obtain the experimental Z-width plots. The control was implemented using SIMULINK and QUARC toolbox on a host PC running Windows XP. The code was compiled and downloaded on a target computer running QNX real-time operating system. The target computer interfaced

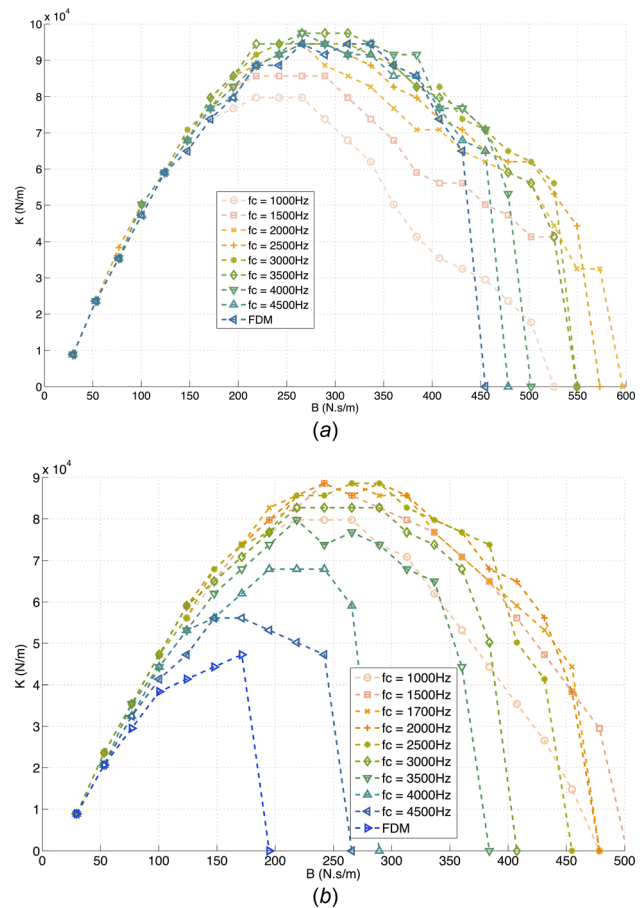


Fig. 4 Z-width boundary plots obtained by simulation of the single-DOF haptic interface device with velocity estimated using FDM + filter. The sampling frequency was kept fixed at 10 kHz and the plots were obtained for varying filter cut-off frequencies (f_c). (a) Position quantization = $1 \mu\text{m}$ and (b) position quantization = $2 \mu\text{m}$.

with the single-DOF haptic interface device through a Q4 data acquisition card from Quanser Inc. The sampling frequency was fixed at 10 kHz and the actuation rate was set at 1 kHz. This allows the velocity estimation and filtering to run at 10 kHz, while actuating at a lower rate to avoid the confounding effects of PWM amplifier switching frequency. The virtual wall is effectively rendered at 1 kHz, but with improved velocity estimation due to higher sampling rate. The Z-width boundaries are plotted for velocity filtered with varying cut-off frequencies. Figures 5(a) and 5(b) show the plots with $1\ \mu\text{m}$ and $2\ \mu\text{m}$ position quantization. The position encoder on the device has a position quantization of $1\ \mu\text{m}$, and for obtaining the plots in Fig. 5(b) the quantization was artificially increased in software.

3 Discussion

Impedance width in a single-DOF haptic interface device is analyzed using three techniques: linear stability analysis, numerical simulation considering the position sensor quantization and actuator saturation nonlinearities, and experimental methods. The goal of these analyses is to observe the effect of velocity filter cut-off frequency on Z-width performance, and investigate if there is an “ideal” cut-off frequency that maximizes the Z-width performance.

Figure 2 shows the variation of uncoupled stability boundary plots obtained with linear analysis. It is observed that with increasing cut-off frequencies, the uncoupled stability boundary

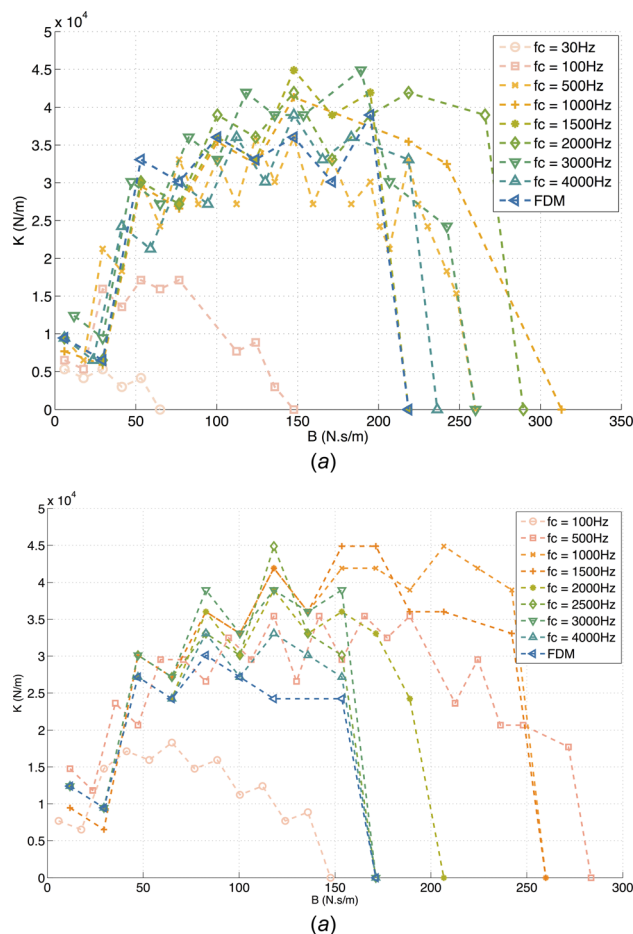


Fig. 5 Experimentally obtained Z-width boundary plots from the single-DOF haptic interface device with velocity estimated using FDM + filter. The sampling frequency was kept fixed at 10 kHz and the plots were obtained for varying filter cut-off frequencies (f_c). (a) Position quantization = $1\ \mu\text{m}$ and (b) position quantization = $2\ \mu\text{m}$.

increases monotonically, which suggests that the Z-width boundary plots might also follow the same trend. The magnitude of the uncoupled stability boundary only gives a theoretical envelope of the Z-width boundary magnitude. The plots obtained with linear analysis do not consider any quantization in position sensing or actuator saturation. To make reasonable predictions about the effect of FDM + filter cut-off frequency on the Z-width performance, a more realistic model of the physical system considering the nonlinearities like position quantization and actuator saturation is needed. A hybrid model incorporating the continuous device dynamics and the discrete sampled and quantized feedback controller would capture the aforementioned nonlinearities, but will not lend itself to analytical stability analysis. Hence, system identification was performed to estimate the physical system parameters, and a numerical simulation incorporating the position quantization and actuator saturation to estimate Z-width performance. The Z-width plots obtained by the simulation are shown in Fig. 4. In most haptics applications, achieving high wall stiffness is more desirable than being able to render high damping. Thus, maximum height of Z-width plots is considered as the determining factor for performance in this study. The simulation plots differ from the plots obtained by linear analysis on several points. First, the magnitudes of the simulation plots are orders of magnitude lower than those obtained by linear analysis. This is expected due to actuator saturation and position quantization effects, which degrade the Z-width performance severely from the ideal case. Second, as observed in Fig. 4(a), the Z-width plots increase in size with increasing FDM + filter cut-off frequency until $f_c = 2000$ Hz, and beyond that no significant increase in Z-width performance is observed. This saturation in Z-width performance was not observed with linear analysis as seen in Fig. 2. These differences between simulation and linear analysis arise from the significant effects of quantization and saturation nonlinearities on the system considered in this paper. Such nonlinearities are common in many haptic interfaces. For a haptic interface with linear dynamics, very low quantization errors in sensing and a range of operation which does not hit actuator saturation limits, a linear analysis might be sufficient to predict an ideal cut-off frequency that maximizes Z-width performance. However, due to the hardware limitations of our device and commonly used haptic interfaces, model based-simulations are needed for a more accurate analysis and a linear analysis would not be sufficient.

Position quantization directly affects the velocity estimation, as any quantization means loss of position information. An increase in quantization will mean that position is held constant for a longer period of time, causing drastic jumps in velocity estimation by FDM. Filtering will smooth down these jumps, and level of smoothing will depend on the choice of cut-off frequency. A higher quantization will require more smoothing, implying use of lower filter cut-off frequencies, which leads to higher phase distortion and degradation of Z-width performance. This hypothesis is validated by comparing the plots in Fig. 4(a) with $1\ \mu\text{m}$ quantization, with plots in Fig. 4(b) with $2\ \mu\text{m}$ quantization. It is observed that with increase in quantization, the saturating trend in Z-width performance observed in Fig. 4(a) changes and a peak Z-width performance is observed at $f_c = 1700$ Hz, after which the Z-width performance starts falling down. The Z-width performance is lower at low cut-off frequencies due to the phase distortion introduced by excessive smoothing by the filter, and degradation of Z-width performance at higher cut-off frequencies is due to passing through of the chatter in velocity signal introduced by the increased quantization. At lower quantization, once the cut-off frequency reached the point where phase distortion by filter smoothing stopped being dominant, the chatter in velocity signal due to quantization was not large enough to cause any significant decrease in Z-width performance with increasing cut-off frequencies.

The experimental results are shown in Fig. 5, which validate the predictions from the simulation results shown in Fig. 4. It is observed in Fig. 5(a) that for a position quantization of $1\ \mu\text{m}$, the

Z-width performance (measured by the height of the Z-width plots) increases with increasing filter cut-off frequencies until saturating at $f_c = 2000$ Hz, as predicted by the simulations in Fig. 4(a). Figure 5(b) shows the Z-width plots when position quantization was set to $2\ \mu\text{m}$. The Z-width performance first increases, and then decreases with increasing filter cut-off frequencies, peaking at $f_c = 1500$ Hz. The experimentally observed peak Z-width performance is close to the value predicted by the simulation results, which is around $f_c = 1700$ Hz as shown in Fig. 4(b). It should be noted that the linear analysis considered a single rate for sensing and actuation (10 kHz) as opposed to the simulations and experiments, which had actuation at 1 kHz and sensing at 10 kHz to be able to estimate better derivatives. Given the inertia of the device (or the model), any high frequency force commands beyond 1 kHz would be redundant and would not make a difference to the results presented.

This study reports successful results in predicting an ideal FDM + filter cut-off frequency that maximizes the Z-width performance of the haptic interface device. Agreement between the experimental and simulated Z-width plots demonstrate that with a reasonably accurate model of the haptic interface, and accounting for quantization and actuator saturation, such an ideal cut-off frequency can be estimated via numerical simulation. Estimating cut-off frequency via numerical simulation will allow control designers to maximize Z-width performance without the time-consuming process of manual tuning based on experiments or the ambiguity involved in ad hoc tuning methods.

4 Conclusion

In this paper, simulation and experimental results are presented to show that there exists an ideal velocity filter cut-off frequency for maximizing the impedance width performance in haptic interfaces. We demonstrated that this ideal cut-off frequency can be predicted by conducting numerical simulations that use a reasonably accurate model of the haptic device and consider the position quantization and actuator saturation nonlinearities. Furthermore, the value of the ideal cut-off frequency depends on the position sensor quantization, and with increase in quantization the ideal filter cut-off frequency and the maximum achievable Z-width performance decrease. It is also shown that the linear stability analysis is not sufficient to predict the effect of velocity filter cut-off frequency on Z-width performance.

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