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A Model Matching Framework for the Synthesis of Series Elastic Actuator Impedance Control

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***Abstract*— The fundamental goal of robot impedance control is to shape a given system's behavior to match that of a predefined desired dynamic model. A variety of techniques are used throughout the literature to achieve this goal, but in practice, most robots ultimately rely on straightforward architectures akin to PD control that have intuitive physical interpretations convenient for the control designer. This is particularly true of systems employing series elastic actuators (SEAs) in spite of the potential that more complex controllers have for improving impedance rendering in devices with higher order dynamics. The model matching framework presented here leverages H_∞ control approaches, that are yet to gain widespread use in the robotics community, to significantly simplify the impedance control design task. This framework provides a novel means by which to synthesize a dynamic feedback controller for an SEA that accommodates a wide range of desired impedances and available feedback. The ease of employing this synthesis approach and its potential benefits for SEA control are discussed in light of the limitations of other existing techniques. This discussion, and the insight gained from a series of simulations comparing impedance controllers designed using established passivity-based techniques to controllers born out of our model matching framework, lay the foundation for further adoption of H_∞ synthesis in SEA control.**

I. Introduction and Background

The use of robotic systems is rapidly expanding beyond traditional automation. Surgery, rehabilitation, human augmentation, and dexterous manipulation for assembly and manufacturing are some of the many applications open to today's robots [1]–[4]. With this exciting growth, it is becoming ever more important for robots to be robust to interactions with their physical environment. Robots are now expected to operate in an unstructured world designed for humans; they must manipulate human tools and interfaces or interact with unmodeled objects, often including humans themselves.

A. Series Elastic Actuation

An increased use of series elastic actuators (SEAs) in the design of robots has paralleled the push for more robust systems. By intentionally incorporating a compliant element in series with the drivetrain of a robotic actuator (shown schematically in Fig. 1), a naturally low output impedance can be provided that is particularly effective at stable interaction with the environment. This stability and improvements in shock tolerance, energy storage, power output, and force sensing are among the many benefits of the SEA architecture widely cited in the literature [5]–[7].

Numerous robotic systems incorporating series elastic actuators have been explicitly designed for interaction with humans or their surrounding environment (e.g. [8], [9]). While individual designs vary, the significant compliance introduced by series elasticity complicates actuator control in most cases. There is a large body of work dating back to the mid 1980's that addresses the control of robots with flexible joints and the need to model actuators as fourth order systems as in Fig. 1 (e.g. [10]–[12]). Most of these studies address the unintentional compliance introduced by gear train flexibility, however, and while the system model used for this case is the exact same as that of an SEA, in practice, an intentional increase in passive compliance requires these past control approaches to be augmented to preserve performance.

B. Impedance Control

Impedance control is a natural approach to take with actuators designed for interaction tasks because it specifically addresses the dynamic relationship between force and velocity at an actuator's output rather than governing these quantities individually [13]. In many ways, impedance control is the virtual equivalent of what series elastic actuation attempts to accomplish physically. By specifying an actuator's desired apparent impedance, one essentially provides a description of how the actuator is to respond to its physical environment.

In "simple" impedance control, as outlined in [13], desired impedances typically take the form of a second order, mass-spring-damper model. This is straightforward to apply to robot manipulators because the controller closely resembles a PD motion controller, where the resulting closed loop stiffness and damping simply correspond to proportional and derivative control gains [14]. Designers can rely on their intuitive understanding of the gains' physical interpretations to quickly generate appropriate control laws and easily iterate designs.

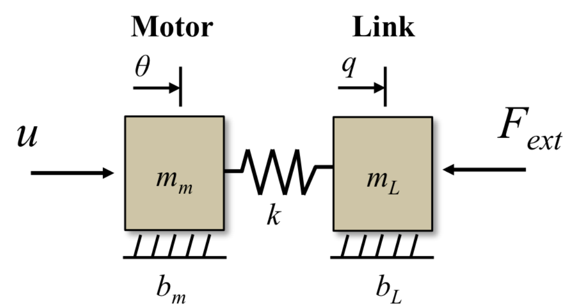

Figure 1. Simplified model of a series elastic actuator. Subscripts m and L represent motor and link variables, respectively. u and F_{ext} are motor and external forces; θ and q are motor and link positions; and m, b, and k are mass, damping, and spring rate, respectively.

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Unfortunately, PD based approaches are less straightforward when the inherent passive dynamics of a series elastic actuator are introduced to the system model. Separating an actuator's motor from its output introduces a choice for the location of sensor feedback. If sensing is colocated with the motor, stability is easily preserved [12]. However, the resulting closed loop impedance in this case is limited in stiffness by the series compliance because the virtual spring of the controller simply acts in series with the passive spring of the actuator (see Fig. 2, for example). Conversely, if actuator output is used exclusively for feedback, stability cannot be guaranteed beyond a small region provided by the passive damping in the actuator [15]. This trade naturally suggests full state feedback as a preferable approach to the impedance control of SEAs or other flexible joint robots, and indeed, various forms of this technique have been applied successfully [16], [17]. As the number of control parameters increases, however, selecting appropriate gains becomes more difficult because their underlying physical meanings are less clear. Thus, controller design is less intuitive.

The passivity-based impedance control of [17] offers an example of attaching meaningful physical interpretations to state feedback gains for flexible joint robots. Specifically, joint torque feedback is interpreted as a means to lower the apparent motor inertia of an actuator. A motor-position-based PD controller can then be applied to generate a desired impedance because decreasing apparent motor inertia serves to make the actuator appear more like a second order system (see Fig. 2). Originally intended for systems with unintentional (i.e. relatively low) gear train compliance, applying this approach more broadly to low stiffness series elastic actuators reveals a few shortcomings. Namely, output stiffness is still limited to the actuator's passive stiffness due, once again, to motor side PD control, and desired output impedances are limited to a second order, mass-spring-damper-like structure to preserve the physical interpretation of the control architecture. As will be shown, the accuracy with which desired impedances are achieved also declines as the actuator's passive stiffness is approached. This is typically not a problem for the stiffer systems of [17] but it does present a real limitation for series elastic actuators, as a stiffness approaching or exceeding passive actuator stiffness is often desired when controlling very soft SEAs.

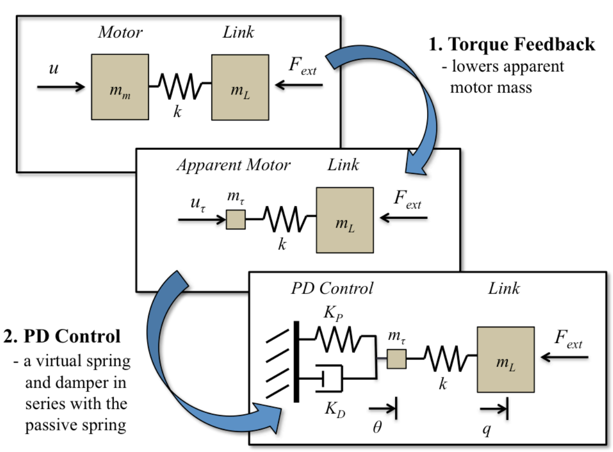

Figure 2. A physical interpretation for state feedback as conceptualized in [17]. Variables are defined as in Fig. 1 with a subscript τ signifying augmentation due to torque feedback.

As outlined in [18], passivity-based impedance control compares favorably to other existing approaches for handling joint flexibility and, in spite of the aforementioned shortcomings, its rigorous stability guarantees make it an attractive option for SEA control as well. Thus, throughout the subsequent sections, impedance control produced using our proposed model matching framework will be benchmarked against the performance of [17]'s control architecture. Recognizing that a full multi-dof application of our synthesis technique, akin to that of [17], is not presented here, this comparison will nevertheless highlight many of the benefits to be gained by incorporating H_∞ control approaches into the design of SEA impedance controllers.

C. H_∞ Control

The goal of H_∞ control is to minimize the response of specified system outputs to specified system inputs as measured by the H_∞ norm of the closed loop transfer function. To achieve this goal, the problem is first formulated in the general control configuration of Fig. 3. Here the objective is to synthesize a controller, K, for the given plant, P, that minimizes the H_∞ norm,

$$\|G\|_\infty = \max_\omega \bar{\sigma}(G(j\omega)) = \max_w \frac{\|z\|_2}{\|w\|_2}, \tag{1}$$

where G is the closed loop transfer function, z is the output vector of interest, and w is the vector of exogenous inputs. This measure, equivalent to the maximum singular value, $\bar{\sigma}$, of the closed loop transfer function, is essentially a ratio of the energy flowing out of the system in response to the energy flowing in. Minimizing $\|G\|_\infty$ has the effect of making the outputs of interest immune (as much as possible) to the exogenous inputs. It should be noted that in the general control configuration, while often desirable, having full state measurements of the plant or direct knowledge of system outputs within the sensor feedback is in no way a prerequisite. In fact, numerous examples in the literature show that almost any linear control problem can be represented by the system in Fig. 3 [19]. Additionally, this construction does not require a predefined physical interpretation of the controller K. Numerous approaches have been used within this framework to synthesize controllers of various structures given a wide range of available sensor feedback.

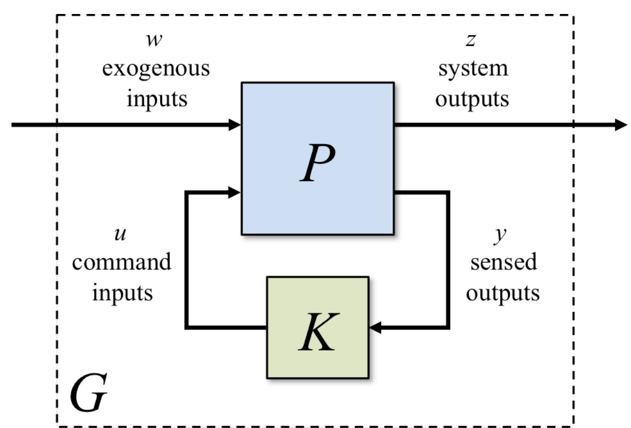

Figure 3. General control configuration.

Synthesizing an H_∞ optimal controller can be accomlished in a number of ways and the framework to be described in the following sections is agnostic in regard to the specific solution method chosen. It should be noted, however, that the H_∞ synthesis problem lends itself to formulation as a set of linear matrix inequality (LMI) constraints, as in [20]. This construction allows for the efficient computation of a solution using readily available software packages like the *MATLAB Robust Control Toolbox* [21] and *cvx* [22]. In fact, leveraging the LMI-based solvers in these software packages within the context of the series elastic actuator impedance control problem allows a designer to synthesize appropriate controller gains through completely automated optimization routines. This enables the application of higher order dynamic controllers to SEA impedance control without the burden of a complex, nonintuitive control design process. Successful controller synthesis is, of course, predicated on formulating the impedance control problem in the appropriate general configuration.

II. Model Matching Framework

A. Problem Formulation

Disturbance rejection is one of the most common uses of H_∞ control because minimizing system response to specific inputs fits naturally into the optimal H_∞ gain framework presented in the previous section. The goal of impedance control, however, is to prescribe, rather than minimize, the relationship between output force and velocity. Thus, to accomplish impedance control, the system output considered in the H_∞ control problem must be reformulated to represent the discrepancy between a desired behavior and the plant's closed loop response. Minimizing the H_∞ norm for this augmented problem will then have the effect of matching system behavior to the specified model.

The model matching framework in Fig. 4 provides an outline for how to construct the SEA impedance control problem to effectively generate controllers via H_∞ synthesis. While the concept of model matching is not new, applying this idea to the impedance control of series elastic actuators is unique in the literature. Furthermore, this framework alleviates many of the deficiencies still outstanding in existing approaches to SEA control design. Principally, after the designer assigns models to each of the blocks in Fig. 4, the selection of controller gains is entirely automated by solving the H_∞ optimization problem. Abstracting the selection of gains away from the control designer eliminates the need for a corresponding intuitive physical interpretation of the controller. Thus, this model matching framework provides a means to synthesize higher order dynamic controllers or address cases where the desired impedance model or available measurement feedback, for example, make it difficult for the designer to approach the problem with an a priori knowledge of proper controller architecture.

Examining the problem formulation in more detail, it can be seen that the plant P from the general configuration of Fig. 3 is augmented in Fig. 4 to now include not only a model of the series elastic actuator, P_{sea}, but also a model of the desired impedance, P_{des}, and a series of weighting functions. The exogenous input, w, is defined as the external force acting on the actuator, F_{ext}, and the output of interest includes the difference between the physical system's position (or velocity) response and that of the desired impedance model.

B. Weighting Functions

The included weighting functions within the model matching framework (represented by W blocks in Fig. 4) allow the control designer to easily customize the synthesis problem to a specific set of physical constraints. Controller effort, u, is included as an additional output in the optimization problem and the weight, W_u, placed on it serves to penalize high motor commands. Thus, the synthesized controller can be constrained to operate within the given actuator's force or torque limitations. W_{ext} and W_{error} serve to bound the synthesis problem by defining magnitude and/or frequency ranges over which the external loads and model matching error, respectively, should be included in the H_∞ norm calculation.

W_{sense} and W_{out} are unique among the included weighting functions because they primarily serve to select signals rather than weight them. Unlike PD control, our method for designing SEA impedance controllers does not presuppose

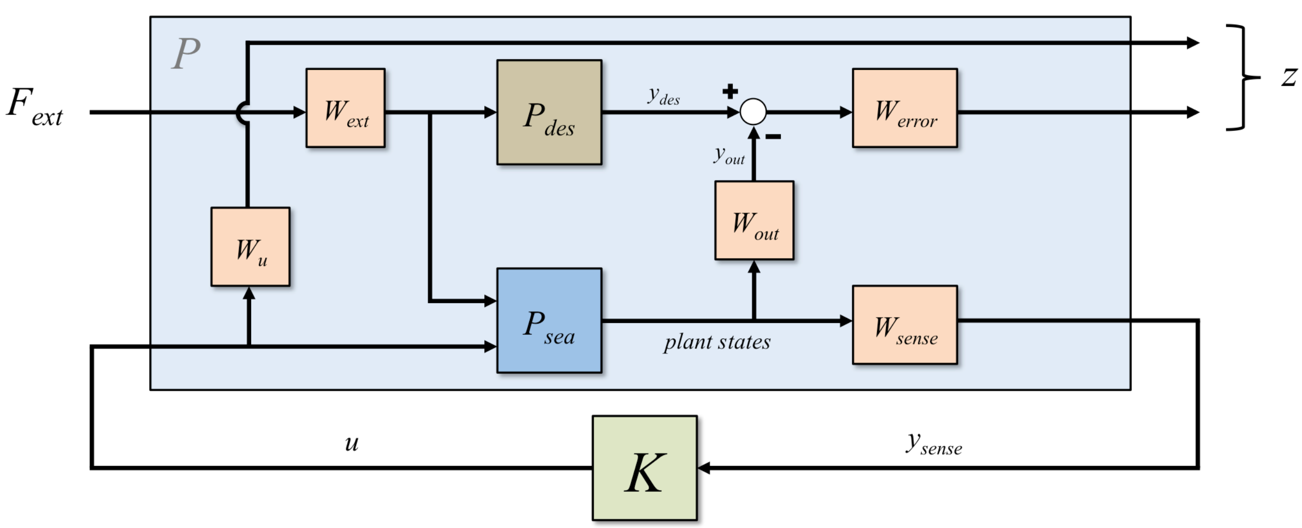

Figure 4. Model matching framework for the synthesis of SEA impedance controllers.

the available sensor feedback. W_{sense} provides a simple matrix for the designer to select which outputs from the physical plant will be available as feedback for the controller. While full state feedback might be ideal for a series elastic actuator, it might not be possible in all cases. Thus, our problem formulation allows for an easy way to modify available sensed outputs without re-architecting the entire system model. This is a valuable tool for empirically studying the feasibility of a desired impedance control scheme in the face of limited sensor data and could perhaps be used in the future to inform actuator design based on the viability of controllers with less feedback.

When varied in conjunction with the structure of the desired impedance model, W_{out} provides the control designer a means to change which actuator states are compared to P_{des} in the model matching operation. The implications of comparing output positions rather than velocities, for instance, are yet to be fully explored. W_{out} allows for this flexibility in problem definition.

An in-depth exploration of the versatility gained by generating impedance control using the presented framework is left for later work. The architecture as constructed in Fig. 4, however, certainly provides a number of options during controller synthesis that are not typically available to the designer given existing techniques.

III. Controller Synthesis

To illustrate the benefits of the proposed model matching framework, the balance of this paper will now examine an example application. A model for the small series elastic actuator presented in [23] will be constructed, and the process of synthesizing appropriate controllers to render a variety of output impedances with this device will be described. The performance of the resulting impedance control will then be compared in simulation to that of the previously discussed passivity-based approach from [17].

A. Model Construction

A state space representation for the series elastic actuator model in Fig. 1 takes the form:

$$P_{sea}\ \begin{cases} \dot{x}_{sea} = A_{sea}x_{sea} + B_{sea\_u}u + B_{sea\_w}w \\ y_{sea} = C_{sea}x_{sea} + D_{sea\_u}u + D_{sea\_w}w \end{cases} \tag{2}$$

where the actuator states, x_{sea}, are simply the position and velocity of each mass. Using the properties determined for the series elastic actuator of [23] (summarized in Table 1), it is straightforward to calculate values for the system matrices A_{sea}, B_{sea_u}, and B_{sea_w}. Additionally, because full state output is desired for the actuator model in our framework, $C_{sea} = I$ and $D_{sea\_u} = D_{sea\_w} = 0$.

For this example, the desired impedance is described as a second order transfer function relating external force input to the actuator's output link position:

$$P_{des}(s) = \frac{1}{m_{des}s^2 + b_{des}s + k_{des}} \ . \tag{3}$$

The values m_{des}, b_{des}, and k_{des} represent the mass, damping, and spring rate of the desired output impedance respectively. It is important to note that the model matching framework does not limit the desired impedance to a second order structure. In fact, controller synthesis can be performed without regard to the order of the desired model. Traditional impedance control techniques could be significantly complicated by higher order desired impedances but H∞ optimization within the model matching framework makes this change almost transparent to the control designer.

Table I. Series Elastic Actuator Properties

Spring Stiffness, k	5500 N/m
Output Viscous Damping, b_L	0 N/(m/s)
Motor Viscous Damping, b_m	10 N/(m/s)
Output Mass, m_L	0.2 kg
Motor Mass, m_m	0.45 kg
Max Actuator Force	22 N

The weighting functions for this specific synthesis problem are selected as follows:

$$W_u = \frac{1}{22} \qquad W_{ext} = 1 \qquad W_{error}(s) = \frac{100k_{des}}{0.1s+1}$$

$$W_{out} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \qquad W_{sense} = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

Of specific note here is the selection of W_{error} as a low pass filter with a 10 rad/s cutoff frequency and a DC gain of 100 times the desired apparent stiffness. This focuses the optimization problem on the low frequency range while enforcing a maximum steady state error as a percentage of the desired deflection due to loading

B. H∞ Synthesis

With all the components of the model matching framework fully defined, the augmented generalized plant, P, can be constructed and the H∞ output feedback problem can be solved. The `hinfsyn` function in the *MATLAB Robust Control Toolbox* is used to solve the set of linear matrix inequality constraints born out of this problem construction, and a dynamic feedback controller is synthesized of the form:

$$K\ \begin{cases} \dot{x}_K = A_K x_K + B_K y_{sense} \\ u = C_K x_K + D_K y_{sense} \end{cases} \tag{4}$$

where x_K represents the states of the dynamic controller. The order of the resulting controller matches that of the augmented plant, P. In this case, $A_K \in \mathbb{R}^{7\times 7}$ because the actuator model is fourth order, the desired impedance model is second order, and an additional state is introduced due to the filter in W_{error}.

IV. Simulations

Once an impedance controller is created, it can be applied directly to the physical series elastic actuator in order to produce an output behavior that matches the desired impedance to within the optimal H∞ norm calculated during synthesis. For subsequent desired impedances, the H∞ optimization problem can be solved again and unique controllers generated. Three such cases for our example actuator are examined here, spanning a range of output impedances. In each case *MATLAB/Simulink* is used to simulate the closed loop behavior of the actuator and plot its response to a 1 N step input in external force.

The first desired impedance tested is considerably softer than the passive SEA, with $m_{des} = 0.2$ kg, $b_{des} = 15$ N/(m/s), and $k_{des} = 300$ N/m. Fig. 5 shows that both passivity-based impedance control and our H_∞ optimal approach provide closed loop step responses very close to the ideal desired model. The dynamic controller synthesized using the model matching framework does show slightly better tracking of the desired model, however.

The benefit of using a model matching approach to synthesize impedance controllers becomes clearer as the desired impedance increases. Fig. 6 plots the step response for each type of controller when asked to render a stiffer, critically damped impedance of $m_{des} = 0.2$ kg, $b_{des} = 49$ N/(m/s), and $k_{des} = 3000$ N/m. While the steady state error is small in both cases, the passivity-based controller exhibits considerable overshoot before settling. Here, the H_∞ optimal dynamic controller appears better equipped than the static state feedback of passivity-based control to render a second order response with the fourth order SEA.

Further increasing the desired impedance with values $m_{des} = 0.2$ kg, $b_{des} = 60$ N/(m/s), and $k_{des} = 4500$ N/m, draws an even clearer distinction between the two approaches to impedance control design. In the step responses of Fig. 7, a discrepancy between the desired impedance and the model matched response begins to be seen, particularly during the initial rise, but this is dwarfed by the large oscillations resulting from the passivity-based controller.

The inability to render a stiffness higher than the passive actuator's is mentioned in [17] as a limit to the passivity-based approach. Degradation in performance as desired stiffness approaches, but does not exceed, passive stiffness is not addressed, however. This is likely not an issue for the relatively stiff systems addressed in [17], but it is problematic for series elastic actuators. An exhaustive analysis of this behavior is not done here, but a remedy within the confines of an intuitive static state feedback controller is not readily apparent. Dynamic feedback controllers synthesized via H_∞ model matching do, however, effectively render the full range of desired impedances tested here. These controllers, as mentioned, are seventh order, meaning that, for this case, the controller in (4) has 72 unique gains that must be determined. This construction is far from intuitive and indeed lacks a clear physical interpretation. However, using an H_∞ based model matching framework preserves the ease of controller construction in spite of increased controller complexity. Automating gain selection makes the use of higher order impedance controllers more realistic in practice. Additionally, H_∞ optimization ensures desired performance can be achieved or, conversely, an infeasible result from the optimization process provides the designer with an easy check on the performance limits of a particular actuator or set of design constraints.

Using the model matching framework to synthesize impedance controllers is not without its own set of shortcomings, however. A high order dynamic controller, with its large number of gains, is considerably more difficult to implement in practice than static state feedback. Systems utilizing this approach will require more processing power and faster control loop rates to perform well. For simple examples like the one presented here, this is certainly not prohibitive, but if the desired impedance model or weighting functions are made higher order, driving the size of the synthesized controller even higher, these practical issues will become even more important to consider. Changing the desired impedance during operation also presents an avenue

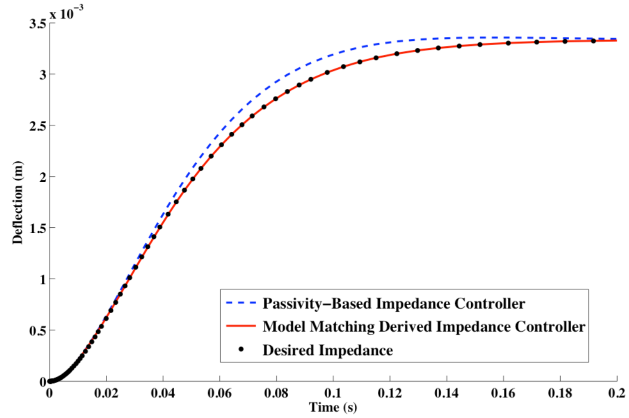

Figure 5. Simulated responses to a 1N step in external force for two controllers rendering $P_{des}(s) = 1/(0.2s^2 + 15s + 300)$.

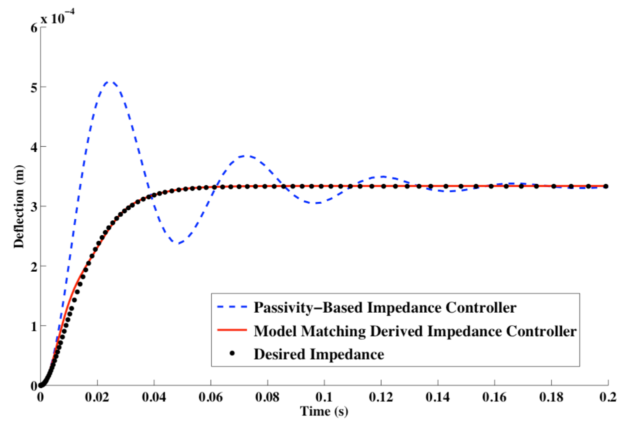

Figure 6. Simulated responses to a 1N step in external force for two controllers rendering $P_{des}(s) = 1/(0.2s^2 + 49s + 3000)$.

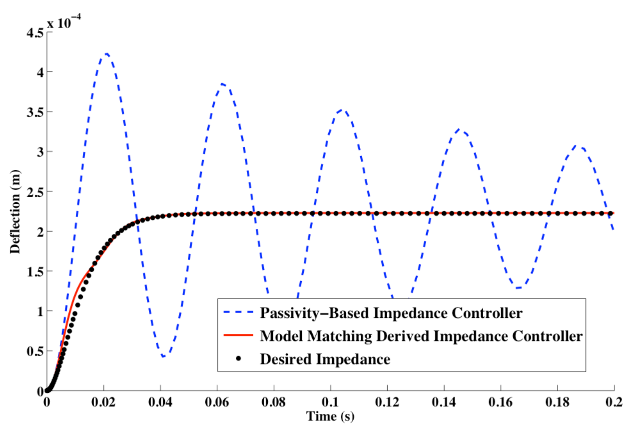

Figure 7. Simulated responses to a 1N step in external force for two controllers rendering $P_{des}(s) = 1/(0.2s^2 + 60s + 4500)$.

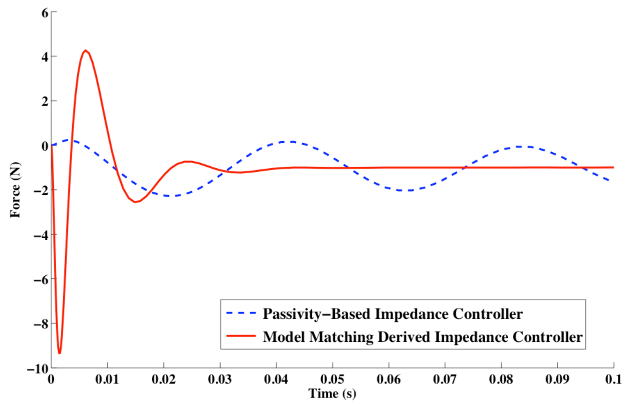

Figure 8. Motor torque required to produce the simulated impedance behaviors in Fig. 7

for further research. This requires solving the optimization problem in real time to provide a new set of controller gains, and additional work is needed to determine the best way to accomplish this in a series elastic robot. The process for constructing appropriate weighting functions within the proposed framework could be refined as well, as this is a task not commonly required for robot impedance control.

It should be noted that the controllers providing improved desired impedance matching here, also require greater actuator effort. Fig. 8 plots the command effort required to achieve the step responses of Fig. 7. While the peak force of the model matching based controller is still well within our actuator's 22N limit, it is significantly higher than that of the passivity-based controller. Greater care in addressing actuator saturation limits is needed when designing within the model matching framework. Because better performance is possible using this design approach though, the designer is free to make intelligent design trades concerning required actuator effort and improved impedance rendering.

V. Conclusion

This work demonstrates the significant promise that H_∞ synthesis has as a means for generating impedance controllers for series elastic actuators. The ability to easily generate control laws that optimally match desired impedances provides a number of benefits over the existing techniques used to design SEA controllers. The model matching framework described here provides an outline for constructing the impedance control design problem to leverage these benefits of H_∞ control.

An extensive discussion on the shortcomings of relying on intuitive physical interpretations for architecting impedance control provides insight into the need for a more versatile, general design process when working with series elastic systems. It is our intention that the model matching framework presented will preserve some of the designer's physical intuition in the design process (through weighting functions for instance) without tying the actual controller gains to this requirement. Thus, more complex controllers can be easily generated in an effort to render higher order desired impedance behaviors, deliver output stiffnesses on the order of (if not greater than) actuator passive stiffness, and provide more rigorous methods of ensuring controller performance across a variety of conditions.

The results of initial simulations suggest that many of the potential benefits of using H_∞ control for SEAs are in fact achievable given the right framework within which to construct the impedance control problem. Much of what has been learned in H_∞ control has yet to be applied to robotics generally, but also more specifically, to the impedance control of robots with flexible joints. This model matching approach to the synthesis of impedance control lays the foundation for further adoption of H_∞ based techniques in the control of systems with series elastic actuators.

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