# On the Performance of Passivity-based Control of Haptic Displays Employing Levant's Differentiator for Velocity Estimation

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**ABSTRACT**

In impedance-type haptic interfaces, encoders are typically employed to provide high resolution position measurements from which velocity is estimated, most commonly via the finite difference method (FDM). This velocity estimation technique performs reliably, unless very fast sampling is required, in which case noise or delay due to filtering of the position signals reduces accuracy in the estimate. Despite this limitation, FDM is attractive because it is a passive process, and therefore the passivity of the overall system can be guaranteed. Levant's differentiator is a viable alternative to FDM, and exhibits increased accuracy in velocity estimation at high sample rates compared to FDM. However, the passivity of this nonlinear velocity estimation technique cannot be shown using conventional methods. In this paper, we employ a time domain passivity framework to analyze and enforce passive behavior of Levant's differentiator for haptic displays in discrete time. The performance of this approach is explored both in simulation and experimentally on a custom made one degree-of-freedom haptic interface. Results demonstrate the effectiveness of the time domain passivity approach for compensating the active behavior observed with use of Levant's differentiator for velocity estimation.

**Keywords:** Dynamic systems and control, Force feedback (kinesthetic) devices, Time domain passivity

## 1 INTRODUCTION

Real-time velocity estimation from position encoder data has been an impediment in improving the performance and ensuring the stability of haptic interfaces. Most real-time velocity estimation schemes share a fundamental trade-off between delay in estimation and noise, neither of which is desirable. In our previous work [3] we proposed the use of Levant's differentiator [6] for velocity estimation with the aim of relaxing this trade-off. Levant's differentiator is a Second Order Sliding Mode (SOSM) control based robust exact differentiation technique.

Levant's differentiator is an attractive choice for real-time velocity estimation due to its desirable characteristics of no delay in estimations and increasing accuracy with increasing sampling rates. On the other hand, like some of the other real-time velocity estimation schemes, passivity properties of Levant's differentiator are yet to be explored and use of Finite Difference Method (FDM) for velocity estimation is widespread due to its passivity properties. With extensive use of passivity based techniques in analyzing the stability properties of haptic interfaces, it is important to study how using Levant's differentiator for velocity estimation affects the passivity property of the system. Replacing velocity estimation using FDM by Levant's differentiator in an otherwise passive haptic interface system may cause loss of passivity. We want to ensure passive behavior along with the benefit of velocity estimation using Levant's differentiator. The differentiator's nonlinear and discontinuous nature precludes the use of conventional techniques like Kalman-Yakubovich-Popov (KYP) lemma and Scattering approach for passivity analysis. Therefore, we use the Time Domain Passivity framework proposed by Hannaford and Ryu in [4] for analyzing the passivity property of the differentiator and enforce passive behavior if and when needed.

The time domain passivity framework has been gaining support in the past decade for passivity analysis and passivity based control of haptic interfaces and teleoperation systems due to attractive features like model-insensitivity and less conservatism than frequency domain passivity approaches [5]. In traditional Time Domain Passivity Approach there is a 'Passivity Observer' (PO) to monitor the energy in real-time and a 'Passivity Controller' (PC) that could dissipate the excess energy determined by the PO in real time. General approach to design a PC is based on the idea of dissipating the excess energy with varying degrees of performance and stability characteristics. A more conservative approach guarantees stability at the cost of degraded performance, for example the one proposed by Ryu et al for teleoperation under time-varying delay in [7]. In contrast, an approach more focussed on improving the performance such as the one proposed by Artigas et al in [1] may not ensure stability under all circumstances. In this study we chose the latter approach which gives more weight to improved performance over guaranteed stability at all times. A $\Delta P$-Passivity Control approach was proposed by Artigas et al in [2] for high fidelity bilateral teleoperation, where the PO monitors the power generated at each sampling period and the PC dissipates that excess energy by modifying one of the output signals. We modified and adapted the $\Delta P$-PC approach for haptic interfaces to compensate the active behavior that may be introduced by estimating velocity using Levant's differentiator. The proposed approach puts a limit on the distortion of the velocity signal thereby retaining a minimum level of haptic performance fidelity but stability is not guaranteed at all times.

The paper is organized as follows. In Section 2, we describe the general framework of the haptic interaction with detailed description on both the use of Levant's differentiator for velocity estimation and the $\Delta P$-Passivity Control approach for compensating the active behavior resulting from the velocity estimation. In Section 3 we describe the simulation model used for testing the proposed approach and Section 4 describes the experimental setup used for experimental validation. In Section 5, we present the simulation and experimental results and discuss the contributions and limitations of the study. Finally we conclude the paper in Section 6.

## 2 METHODS

In order to illustrate the role of velocity estimation, we first describe the general single Degree of Freedom (DOF) haptic interface model and then discuss application of Levant's differentiator for veloc-

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ity estimation. Then, the $\Delta P$ passivity controller is presented with the modified single-DOF haptic interface schematic to compensate for the active behavior resulting from the velocity estimation using Levant's differentiator. Finally, the implementation of the proposed approach in simulation and experiment on a single-DOF haptic device is described.

Consider a typical single DOF haptic interface device interacting with a virtual environment as shown by the model in Fig. 1. We consider the case where we have access to the position data only and velocity is estimated by taking the derivative of the position signal in real-time. $Z_h$ and $Y_{HI}$ are the transfer functions of the human operator and the haptic interface respectively. We assume that the human operator, the haptic interface and the virtual environment are passive. Human operator forms a closed loop with haptic interface

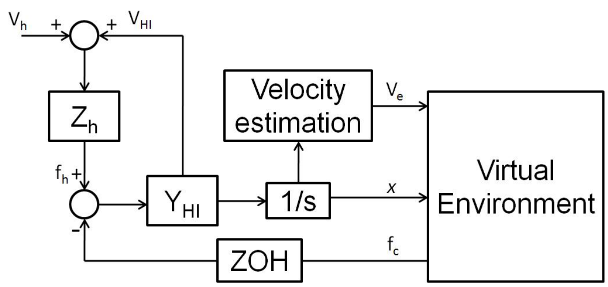



Figure 1: General schematic of a single-DOF haptic interface.

and the haptic interface is in a closed loop with the virtual environment. Levant's differentiator is used to estimate velocity from the position encoder signal, and is described in the following section.

### 2.1 Levant's Differentiator

Levant's differentiator is essentially a nonlinear observer for a chain of integrators driven with $\ddot{f}(t)$ being viewed as the disturbance. Levant proposed a robust exact differentiation technique based upon 2-sliding algorithm for signals with a given upper bound on the Lipschitz constant of the derivative [6]. Given an input signal $f(t)$, the Lipschitz constant of the derivative is a constant $C$ which satisfies

$$\left|\dot{f}(t_1) - \dot{f}(t_2)\right| \le C\left|t_1 - t_2\right| \tag{1}$$

If the second derivative of the base signal exists, then the Lipschitz constant in equation (1) satisfies

$$\sup_{t \ge t_0}\left|\frac{d^2}{dt^2} f(t)\right| \le C \tag{2}$$

where $t_0$ is the initial time.

In order to differentiate the unknown base signal, consider the auxiliary equation

$$\dot{x} = u \tag{3}$$

In the following equations, it is assumed that $f, x, u_1$ are measured at discrete times with time interval $\tau$ and let $t_i, t, t_{i+1}$ be successive measuring times with $t \in [t_i, t_{i+1}]$. We then define $e(t) = x(t) - f(t)$ and in order to have $u$ as the derivative of the input signal $f(t)$, the following 2-sliding algorithm is applied to ensure $e = 0$

$$u = u_1(t_i) - \lambda\left|e(t_i)\right|^{1/2} sign(e(t_i)) \tag{4}$$

$$\dot{u}_1 = -\alpha sign(e(t_i)) \tag{5}$$

Here $u(t)$ is the output of the differentiator and solutions of the system described by equations (3), (4) and (5) are understood in the sense of Filippov. Gains $\lambda$ and $\alpha$ are strictly positive constants which determine the differentiation accuracy and must be chosen properly to ensure convergence. Levant proposed a *sufficient* condition for the convergence of $u(t)$ to $\dot{f}(t)$ given as

$$\alpha > C\ ,\ \lambda^2 \ge 4C\frac{\alpha + C}{\alpha - C} \tag{6}$$

An easier choice of the parameters given in the same reference is

$$\alpha = 1.1C\ ,\ \lambda = C^{1/2} \tag{7}$$

It should be noted that conditions (6) and (7) result from a very crude estimation of the convergence criterion. It is possible to choose a pair of gains $\alpha$ and $\lambda$ that fail to satisfy the conditions (6) or (7), yet still result in stable behavior of the differentiator. It should be noted that the Levant's differentiator is most effective at high sampling rates ($\geq$ 10 kHz)

For further insight into velocity observers using a chain of integrators, reader is referred to [8] where a high gain observer, which is a linear Luenberger observer for a chain of integrators, and the Levant's differentiator are compared. The position signal is an input to the observer, which then estimates the velocity by driving the error between observed position and the estimated position to zero. In the case of the high gain observer, the control law driving the error to zero is linear and in the case of Levant's differentiator, its nonlinear.

### 2.2 $\Delta P$ Passivity Controller

The role of the $\Delta P$-Passivity Controller (PC) is to prevent any energy leaks due to active behavior of the velocity estimation block while not causing excessive distortion of the velocity signal. Fig. 2 shows the block diagram with the modified $\Delta P$-PC in place. The position signal is directly passed to the virtual environment, and since the dynamics of the haptic device is passive, no active behavior is expected to result from this.

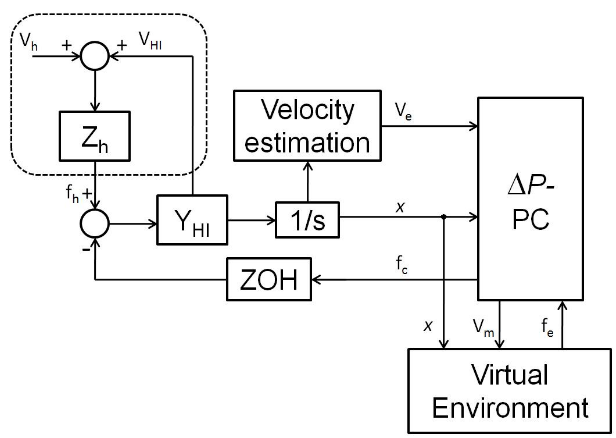



Figure 2: Schematic of a single-DOF haptic interface with modified $\Delta P$ passivity controller to compensate for the active behavior due to velocity estimation.

The $\Delta P$-PC has three inputs, the estimated velocity ($v_e$), position data from the haptic interface ($x$) and the force commanded by the virtual environment ($f_e$). It has two outputs, the modified velocity signal to the virtual environment ($v_m$) and the force signal going to the haptic interface ($f_c$). The $\Delta P$-PC computes the power generated or dissipated at every sampling period. A correction term for the velocity signal is then computed to compensate for any excess power generated or dissipated, thereby plugging any energy leaks. In a deviation from the $\Delta P$ approach proposed by Artigas et al [2] for bilateral teleoperation, we allow the correction term to have both negative and positive values since not only generation of energy, but also accumulation of energy is undesirable as such accumulation leads to more conservative behavior of the system. The purpose of the $\Delta P$-PC is to bring $\Delta P$, and hence $\Delta E$, to zero. This is achieved by first computing $\Delta E$, the net energy generated or dissipated in a sampling period $T_s$, using

$$\Delta E(k) = (x(k+1) - x(k)) f_c(k) - T_s f_e(k) v_m(k) \tag{8}$$

Starting from $E(0) = 0$, the net energy stored in the $\Delta P$-PC block is given by

$$E(k+1) = E(k) + \Delta E(k) \tag{9}$$

The power generated in a sampling period $T_s$ is given by $\Delta P(k) = \Delta E(k)/T_s$ and the correction factor for velocity is computed as:

$$v_c(k) = \Delta P(k)/f_e(k) \tag{10}$$

The outputs are chosen as:

$$f_c(k) = f_e(k) \tag{11}$$

$$v_m(k+1) = v_e(k+1) + v_c(k) \tag{12}$$

We put a maximum bound on the absolute value of the correction term such that the velocity signal is not excessively distorted.

$$|v_c| \leq \delta v \tag{13}$$

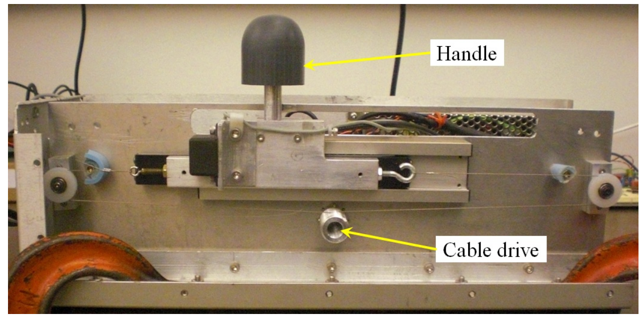
While this bounding operation limits the distortion of the velocity signal, we do allow, to some extent, power generation in the $\Delta P$-PC block.

## 3 Simulations

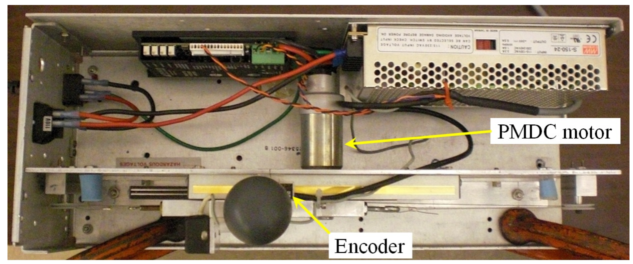
Simulation is used to demonstrate the efficacy of the proposed modified $\Delta P$-PC approach for compensating active behavior resulting from the velocity estimation. For simulating the haptic interaction with a virtual wall using a single-DOF haptic interface device, the human operator is taken out of the loop as we want to test the performance of $\Delta P$-PC in a more controlled manner. We simulate an automated wall hit where we replace $f_h$, the force applied by human operator by a constant force input as shown by the dotted line in Fig. 2. Matlab and Simulink are used for the simulations. The haptic interface is modeled as a mass-damper system with mass $M_{HI} = 0.1kg$ and coefficient of viscous friction $B_{HI} = 0.5N.s/m$. The virtual environment is simulated as a spring-damper virtual wall with virtual wall stiffness $K_e = 10000N/m$ and virtual wall damping $B_e = 10N.s/m$. Gains for Levant's differentiator were chosen as $\alpha = 12m/s^2$ and $\lambda = 5m/s^2$. The simulation was run at a loop rate of 10kHz.

## 4 Experiments

Experiments are conducted to validate the results obtained from the simulation. The experimental setup consists of a one degree-of-freedom custom built linear haptic device that displays forces on a palm grip handle, as shown in Fig. 3. The forces displayed on the handle are proportional to the current applied to the permanent magnet DC motor (Faulhaber, 3557K024C) driving the handle assembly which translates on a ball-slider (Del-Tron Precision Inc., model S2-6). The voltage output of the DAC (digital-to-analog convertor) is passed through a pulse width modulation (PWM) amplifier (Advanced Motion Controls) operating in current mode to drive the motor. A micrometer precision position encoder (Renishaw, RGH24X) is mounted on the handle assembly to accurately measure the handle position. The haptic interface has a workspace of approximately 0.15 m and a maximum continuous force output of 4 N. The bandwidth of the device is determined to be 30 Hz.




(a) Front view




(b) Top view

Figure 3: A single degree-of-freedom haptic device is used as the experimental setup.

Simulink with QuaRC was used to implement the control on a host computer running Windows. The code is compiled and downloaded on a target computer running QNX real-time operating system for hard real-time implementation. The target computer is interfaced to the haptic device through a Q4 data acquisition board from Quanser Inc. The loop rate was set at 10 kHz. Similar to the automated wall hitting protocol used in [3], we take the human operator out of the loop and instead apply a constant force $f_h$ as shown in Fig. 2.

## 5 Results and Discussion

Figures 4 and 5 show the results obtained by simulation of the single-DOF haptic device interacting with a virtual wall and velocity estimated by Levant's differentiator. The position of the haptic interface and the energy accumulated in the $\Delta P$-PC block are plotted with time. We chose a pair of gains $\alpha$ and $\lambda$ for Levant's differentiator such that we have an unstable wall interaction, as evident by the presence of high frequency oscillations as seen in Fig. 4. After switching on the $\Delta P$-PC, we observe that the haptic interaction is stable and free of high frequency oscillations. Looking at the energy plot in Fig. 5, we can observe the active behavior associated with the unstable interaction. With the $\Delta P$-PC switched on, there is some active behavior in the beginning when the controller is trying to compensate for the energy leak but is not able to because of the limit on the distortion of the velocity signal. The $\Delta P$-PC manages to compensate for the energy leak within 0.3 seconds and we see that the total accumulated energy reaches a constant value. The final value of energy accumulated is negative which can be attributed to the energy generated during the initial active phase.

The $\Delta P$-PC approach was implemented experimentally on the linear single-DOF haptic device. The wall hit was automated by replacing the force applied by the human operator with a constant input force. The gains for the Levant's differentiator were chosen to be $\alpha = 13000m/s^2$ and $\lambda = 50m/s^2$. Figures 6 and 7 show the results obtained from the experimental implementation. Fig. 6 shows the position of the handle plotted against time for the $\Delta P$-PC switched off and on cases. With the $\Delta P$-PC off, unstable wall in-

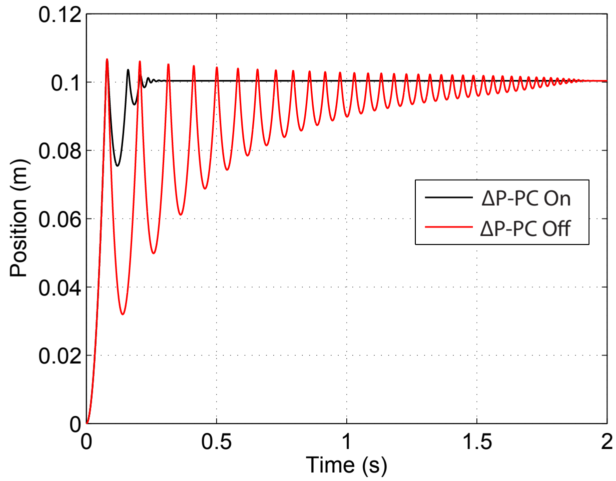



Figure 4: Simulation plot of position vs. time with $\Delta P$-PC on and off.

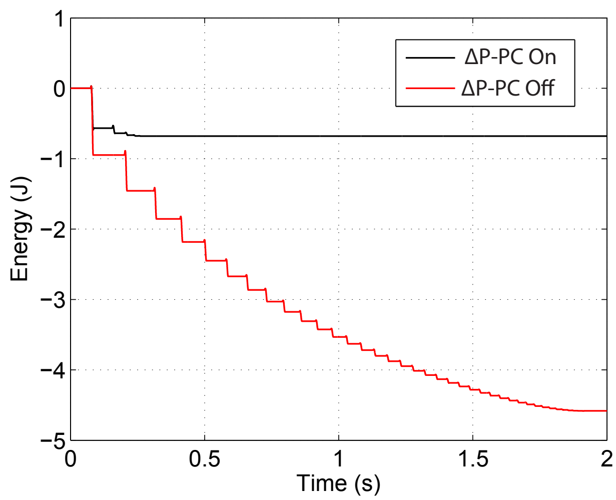



Figure 5: Simulation plot of energy accumulated in the $\Delta P$-PC block vs. time with $\Delta P$-PC on and off.

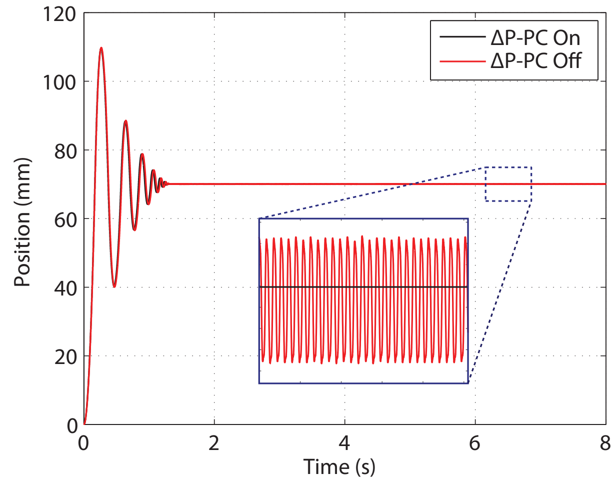



Figure 6: Experimental plot of handle position vs. time with $\Delta P$-PC on and off.

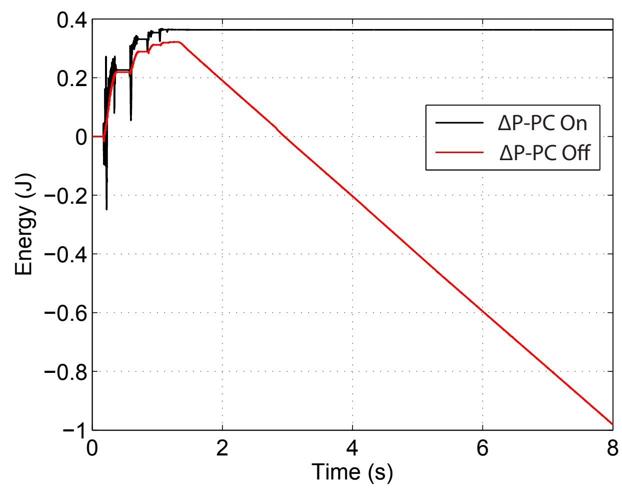



Figure 7: Experimental plot of energy accumulated in the $\Delta P$-PC block vs. time with $\Delta P$-PC on and off.

teraction was observed with high frequency oscillations, as shown in the call-out. Although the magnitude of these sustained oscillations is small with respect to the range of motion, a distinct sound associated with the high frequency oscillations can be observed in the video attachment. Some chatter in the energy plot in Fig. 7 with the $\Delta P$-PC switched on is observed, which happens during the period that the handle is interacting with the virtual wall and the PC is trying to resist any change in energy and bring $\Delta E$ to zero. The simulation and experimental implementations differ due to limitations in the hardware. The system is incapable of displaying stiffnesses large enough to generate the bouncing behavior observed in the simulation plot.

This study reports successful results from the implementation of the $\Delta P$-PC approach to compensate for the active behavior observed with use of Levant's differentiator for velocity estimation in haptic interactions.

One limitation of the proposed approach is the fact that it cannot guarantee stability at all times. We allow for some active behavior by limiting the amount of distortion permitted for the velocity signal. In the current implementation, this upper bound on the velocity correction factor $\delta v$ is fixed, and its choice is dependent on the application. It should be noted that $\Delta P$-PC can only enforce discrete-time passive behavior. Another limitation is that performance of the Levant's differentiator is dependent on choice of the gains $\alpha$ and $\lambda$. The gains are a function of the signal characteristics such that low and high velocity signals may require different sets of optimized gains. Finally, for achieving best performance out of the Levant's differentiator, high sampling rates are recommended ($\geq$ 10 kHz) which may require the use of specialized hardware.

Further analysis is warranted, for example we have not studied the effect of the proposed approach on the Z-width in the haptic interface. By making the unstable interactions stable, we expect to achieve a higher Z-width, but the errors in velocity estimation may get amplified at higher virtual damping values and limit the range of the Z-width boundary. Investigation of the effect of the choice of $\delta v$ on the Z-width may give some insight on optimal selection of $\delta v$ in order to achieve best performance with minimal distortion of the velocity signal.

## 6 Conclusion

In this paper, we proposed a $\Delta P$-Passivity Control approach to compensate the active behavior of Levant's differentiator used for velocity estimation in haptic interaction. By using the proposed approach, it was possible to compensate the active behavior associated with the velocity estimation. Simulation and experimental results were provided to demonstrate the efficacy of the approach. Future work will involve studying the effect of the proposed approach on the Z-width in haptic interfaces.

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